

## Contents

<u>CHAPTER I. BASIC DEFINITIONS</u>	1
<u>§ 1. Some elementary facts</u>	1
Process (2*), measurable (3), indistinguishable (4), predictable (5). Martingale (6-8). Stopping (9-14). Transforms (15). DOOB's decomposition (16-20). LEVY's BOREL- CANTELLI lemma (21-22).	
<u>§ 2. Predictability in continuous time</u>	10
Generators for the basic fields (23-28). Predictable stopping (29-30). The section theorem (31-33). Fields $\mathbb{F}_T$ (34-37). Increasing processes (38-45).	
 <u>CHAPTER II. DISCRETE MARTINGALE THEORY</u>	24
<u>§ 1. Classical theory: sampling, inequalities, convergence</u>	24
Optional sampling (bounded stopping times) (1-3). The maximal inequality (4-5) and its conditioning (6). DOOB's $L^p$ bounds (7-11). Convergence: CHATTERJI's theorem (12-14), real case (15), KRICKEBERG decomposition (16), final result (17-18). Convergence: the upcrossing method (19-23). Convergence, infinity to the left (24-26). Optional sampling (27-31). RIESZ decompositions (32-33), potentials (34-35). A curious result on $X_n^*$ (36).	
<u>§ 2. A sample of modern martingale theory</u>	45
Class (D) processes (37-40). Local supermartingales (41-44). Estimating DOOB's decomposition: bounded case (45-46). Application: BURKHOLDER's maximal lemma for transforms (47) and $L^p$ bounds (48). DOOB's decomposition, general case: maximal inequalities (49-50), tame convex functions (51)	

\* All definitions, theorems, remarks ... are sequentially numbered in each chapter. References throughout the book, and in this table, concern these numbers (not page numbers).

and NEVEU's lemma (52), application to DOOB's decomposition (52') and to the BURKHOLDER-DAVIS-GUNDY theorem (increasing processes) (53). Quadratic variation and variance process (54-58). The  $L^p$  inequalities of BURKHOLDER (60) [RADEMACHER functions, KHINCHIN's lemma, (59)]. Generalization of the BOREL-CANTELLI lemma and the strong law of large numbers (61-66). Martingales with bounded jumps, exponential bounds (67-70). Partial proof of the law of the iterated logarithm (71-72).

<u>APPENDIX 1.</u> DOOB's original proof of the convergence theorem . . . . .	74
<u>APPENDIX 2.</u> Another proof of the BURKHOLDER $L^p$ inequalities, and DAVIS's theorem . . . . .	75
<u>APPENDIX 3.</u> Some complements to BURKHOLDER's inequalities . . . . .	83
Index of notations and definitions . . . . .	85
Bibliography . . . . .	88