

# Table of Contents

|   |    |
|---|----|
| Introduction .....  | 1  |
| Notation .....  | 7  |
| <b>Part 0. Isoperimetric Background and Generalities</b>                                      |    |
| Chapter 1. Isoperimetric Inequalities and the Concentration<br>of Measure Phenomenon .....    | 14 |
| 1.1 Some Isoperimetric Inequalities on the Sphere, in Gauss Space<br>and on the Cube .....    | 15 |
| 1.2 An Isoperimetric Inequality for Product Measures .....                                    | 25 |
| 1.3 Martingale Inequalities .....   | 30 |
| Notes and References .....  | 34 |
| Chapter 2. Generalities on Banach Space Valued Random Variables<br>and Random Processes ..... | 37 |
| 2.1 Banach Space Valued Radon Random Variables .....  | 37 |
| 2.2 Random Processes and Vector Valued Random Variables .....                                 | 43 |
| 2.3 Symmetric Random Variables and Lévy's Inequalities .....                                  | 47 |
| 2.4 Some Inequalities for Real Valued Random Variables .....                                  | 50 |
| Notes and References .....  | 52 |
| <b>Part I. Banach Space Valued Random Variables<br/>and Their Strong Limiting Properties</b>  |    |
| Chapter 3. Gaussian Random Variables .....  | 54 |
| 3.1 Integrability and Tail Behavior .....   | 56 |
| 3.2 Integrability of Gaussian Chaos .....   | 64 |
| 3.3 Comparison Theorems .....   | 73 |
| Notes and References .....  | 87 |

|  |         |
|--|---------|
| Chapter 4. Rademacher Averages .....   | 89      |
| 4.1 Real Rademacher Averages .....   | 89      |
| 4.2 The Contraction Principle .....  | 95      |
| 4.3 Integrability and Tail Behavior of Rademacher Series .....   | 98      |
| 4.4 Integrability of Rademacher Chaos .....  | 104     |
| 4.5 Comparison Theorems .....  | 111     |
| Notes and References .....   | 120     |
| <br>Chapter 5. Stable Random Variables .....   | <br>122 |
| 5.1 Representation of Stable Random Variables .....  | 124     |
| 5.2 Integrability and Tail Behavior .....  | 133     |
| 5.3 Comparison Theorems .....  | 141     |
| Notes and References .....   | 147     |
| <br>Chapter 6. Sums of Independent Random Variables .....  | <br>149 |
| 6.1 Symmetrization and Some Inequalities for Sums of<br>Independent Random Variables .....             | 150     |
| 6.2 Integrability of Sums of Independent Random Variables .....  | 155     |
| 6.3 Concentration and Tail Behavior .....  | 162     |
| Notes and References .....   | 176     |
| <br>Chapter 7. The Strong Law of Large Numbers .....   | <br>178 |
| 7.1 A General Statement for Strong Limit Theorems .....  | 179     |
| 7.2 Examples of Laws of Large Numbers .....  | 186     |
| Notes and References .....   | 195     |
| <br>Chapter 8. The Law of the Iterated Logarithm .....   | <br>196 |
| 8.1 Kolmogorov's Law of the Iterated Logarithm .....   | 196     |
| 8.2 Hartman-Wintner-Strassen's Law of the Iterated Logarithm .....                                     | 203     |
| 8.3 On the Identification of the Limits .....  | 216     |
| Notes and References .....   | 233     |
| <br><b>Part II. Tightness of Vector Valued Random Variables<br/>and Regularity of Random Processes</b> |         |
| <br>Chapter 9. Type and Cotype of Banach Spaces .....  | <br>236 |
| 9.1 $\ell_p^n$ -Subspaces of Banach Spaces .....   | 237     |
| 9.2 Type and Cotype .....  | 245     |
| 9.3 Some Probabilistic Statements in Presence<br>of Type and Cotype .....                              | 254     |
| Notes and References .....   | 269     |

|   |     |
|---|-----|
| Chapter 10. The Central Limit Theorem .....   | 272 |
| 10.1 Some General Facts About the Central Limit Theorem .....                             | 273 |
| 10.2 Some Central Limit Theorems in Certain Banach Spaces .....                           | 280 |
| 10.3 A Small Ball Criterion for the Central Limit Theorem .....                           | 289 |
| Notes and References .....  | 295 |
| Chapter 11. Regularity of Random Processes .....  | 297 |
| 11.1 Regularity of Random Processes Under Metric<br>Entropy Conditions .....              | 299 |
| 11.2 Regularity of Random Processes Under Majorizing<br>Measure Conditions .....          | 309 |
| 11.3 Examples of Applications .....   | 318 |
| Notes and References .....  | 329 |
| Chapter 12. Regularity of Gaussian and Stable Processes .....                             | 332 |
| 12.1 Regularity of Gaussian Processes .....   | 333 |
| 12.2 Necessary Conditions for the Boundedness and<br>Continuity of Stable Processes ..... | 349 |
| 12.3 Applications and Conjectures on Rademacher Processes .....                           | 357 |
| Notes and References .....  | 363 |
| Chapter 13. Stationary Processes and Random Fourier Series .....                          | 365 |
| 13.1 Stationarity and Entropy .....   | 365 |
| 13.2 Random Fourier Series .....  | 369 |
| 13.3 Stable Random Fourier Series and<br>Strongly Stationary Processes .....              | 382 |
| 13.4 Vector Valued Random Fourier Series .....  | 387 |
| Notes and References .....  | 392 |
| Chapter 14. Empirical Process Methods in Probability<br>in Banach Spaces .....            | 394 |
| 14.1 The Central Limit Theorem for Lipschitz Processes .....                              | 395 |
| 14.2 Empirical Processes and Random Geometry .....  | 402 |
| 14.3 Vapnik-Chervonenkis Classes of Sets .....  | 411 |
| Notes and References .....  | 419 |
| Chapter 15. Applications to Banach Space Theory .....                                     | 421 |
| 15.1 Subspaces of Small Codimension .....   | 421 |
| 15.2 Conjectures on Sudakov's Minoration for Chaos .....                                  | 427 |
| 15.3 An Inequality of J. Bourgain .....   | 430 |
| 15.4 Invertibility of Submatrices .....   | 434 |

|  |     |
|--|-----|
| 15.5 Embedding Subspaces of $L_p$ into $\ell_p^N$ .....                          | 438 |
| 15.6 Majorizing Measures on Ellipsoids .....                                     | 448 |
| 15.7 Cotype of the Canonical Injection $\ell_\infty^N \rightarrow L_{2,1}$ ..... | 453 |
| 15.8 Miscellaneous Problems .....  | 456 |
| Notes and References .....   | 459 |
| References .....   | 461 |
| Subject Index .....  | 478 |