

Contents

Introduction	1
1 Probability theory on simply connected nilpotent Lie groups	7
1.1 Continuous convolution semigroups of probability measures	7
1.2 Limit theorems	15
1.3 Some potential theory	21
2 Brownian motions on \mathbb{H}	29
2.1 Weak convergence to Brownian motion	29
2.1.1 Uniqueness of embedding of Gaussian measures	29
2.1.2 The Lindeberg and Ljapunov theorems	31
2.1.3 The domain of normal attraction	37
2.1.4 A Gaussian central limit theorem for intermediately trimmed products	39
2.2 Capacities	44
2.2.1 The Wiener sausage	44
2.2.2 Transience, recurrence, and the Lebesgue needle	54
2.3 A.s. results	58
2.3.1 Local and asymptotic behavior of Brownian motion	58
2.3.2 The Crépel-Royette law of the iterated logarithm	73
2.3.3 The subsequence principle	82
3 Other limit theorems on \mathbb{H}	85
3.1 Weak theorems	85
3.1.1 Universal laws	85
3.1.2 Domains of attraction of stable semigroups	86
3.1.3 Lightly trimmed products	87
3.1.4 Tutubalin's theorem	96
3.1.5 Triangular systems	97
3.2 A.s. theorems	101
3.2.1 The Marcinkiewicz-Zygmund law of large numbers	101
3.2.2 Rates of convergence in laws of large numbers	106
3.2.3 The ergodic theorem	111
3.2.4 Non-classical laws of the iterated logarithm	112
3.2.5 The two-series theorem	121

Bibliography

125

Index

137