

Contents

Preface	xv
List of Tables	xix
1 Preliminary Information	1
Introduction, 1	
A Mathematical Preliminaries, 1	
A1 Factorial and Combinatorial Conventions, 1	
A2 Gamma and Beta Functions, 4	
A3 Finite Difference Calculus, 8	
A4 Differential Calculus, 12	
A5 Incomplete Gamma and Beta Functions, and Other Gamma-Related Functions, 14	
A6 Gaussian Hypergeometric Functions, 17	
A7 Confluent Hypergeometric Functions (Kummer's Functions), 19	
A8 Generalized Hypergeometric Functions, 22	
A9 Bernoulli and Euler Numbers and Polynomials, 25	
A10 Integral Transforms, 27	
A11 Orthogonal Polynomials, 27	
A12 Miscellaneous Topics, 29	
B Probability and Statistical Preliminaries, 32	
B1 Calculus of Probabilities, 32	
B2 Real Variables, 35	
B3 Bayes' Theorem, 38	
B4 Expected Values, 39	
B5 Moments and Moment Generating Functions, 40	
B6 Cumulants and Cumulant Generating Functions, 44	

B7	Joint Moments and Cumulants,	46
B8	Characteristic Functions,	47
B9	Probability Generating Functions,	48
B10	Order Statistics,	51
B11	Truncation and Censoring,	52
B12	Mixture Distributions,	53
B13	Variance of a Function,	54
B14	Geometrical Concepts,	55
B15	Inference,	56
C	Computer Generation of Univariate Discrete Random Variables,	61
C1	General Comments,	61
C2	Distributionally Non-Specific Generation Procedures,	62
C3	Binomial Random Variables,	64
C4	Poisson Random Variables,	65
C5	Negative Binomial Random Variables,	66
C6	Hypergeometric Random Variables,	67
C7	Logarithmic Random Variables,	68

2	Families of Discrete Distributions	69
1	Lattice Distributions,	69
2	Power Series Distributions,	70
2.1	Generalized Power Series Distributions,	70
2.2	Modified Power Series Distributions,	74
3	Difference Equation Systems,	77
3.1	Katz and Extended Katz Families,	77
3.2	Ord's Family,	81
4	Kemp Families,	84
4.1	Generalized Hypergeometric Probability Distributions,	84
4.2	Generalized Hypergeometric Factorial Moment Distributions,	91
5	Distributions Based on Lagrangian Expansions,	95
5.1	Otter's Multiplicative Process,	95
5.2	Lagrangian Distributions,	96
5.3	Gould and Abel Distributions,	100
6	Factorial Series Distributions,	102
3	Binomial Distribution	105
1	Definition,	105
2	Historical Remarks and Genesis,	106

3	Moments, 106	
4	Properties, 109	
5	Order Statistics, 113	
6	Approximations, Bounds, and Transformations, 114	
6.1	Approximations, 114	
6.2	Bounds, 120	
6.3	Transformations, 122	
7	Computation and Tables, 123	
8	Estimation, 124	
8.1	Model Selection, 124	
8.2	Point Estimation, 125	
8.3	Confidence Intervals, 129	
8.4	Model Verification, 132	
9	Characterizations, 133	
10	Applications, 134	
11	Truncated Binomial Distributions, 135	
12	Other Related Distributions, 137	
12.1	Limiting Forms, 137	
12.2	Poissonian Binomial, Lexian, and Coolidge Schemes, 138	
12.3	Binomial-Binomial Lagrangian Distributions, 142	
12.4	Weighted Binomial Distributions, 145	
12.5	Pseudo-Binomial Variables, 147	
12.6	Correlated Binomial Variables, 148	
4	Poisson Distribution	151
1	Definition, 151	
2	Historical Remarks and Genesis, 151	
3	Moments, 156	
4	Properties, 158	
5	Approximations, Bounds, and Transformations, 162	
6	Computation and Tables, 165	
7	Estimation, 166	
7.1	Model Selection, 166	
7.2	Point Estimation, 168	
7.3	Confidence Intervals, 170	
7.4	Model Verification, 172	
8	Characterizations, 173	
9	Applications, 179	
10	Truncated and Misrecorded Poisson Distributions, 181	

10.1	Left-Truncation, 181	
10.2	Right-Truncation and Double-Truncation, 184	
10.3	Misrecorded Poisson Distributions, 186	
11	Poisson-Stopped-Sum Distributions, 188	
12	Other Related Distributions, 189	
12.1	The Normal Distribution, 189	
12.2	The Gamma Distribution, 189	
12.3	Sums and Differences of Poisson Variates, 190	
12.4	Hyper-Poisson Distributions, 192	
12.5	Grouped Poisson Distributions, 195	
12.6	Heine and Euler Distributions, 197	
5	Negative Binomial Distribution	199
1	Definition, 199	
2	Geometric Distribution, 201	
3	Historical Remarks and Genesis, 203	
4	Moments, 207	
5	Properties, 208	
6	Approximations and Transformations, 209	
7	Computation and Tables, 213	
8	Estimation, 214	
8.1	Model Selection, 214	
8.2	P Unknown, 215	
8.3	Both Parameters Unknown, 216	
8.4	Data Sets with a Common Parameter, 219	
9	Characterizations, 220	
9.1	Geometric Distribution, 220	
9.2	Negative Binomial Distribution, 223	
10	Applications, 223	
11	Truncated Negative Binomial Distributions, 225	
12	Other Related Distributions, 228	
12.1	Limiting Forms, 228	
12.2	Engen's Extended Negative Binomial Model, 229	
12.3	Lagrangian "Generalized Negative Binomial Distribution", 230	
12.4	Weighted Negative Binomial Distributions, 230	
12.5	Convolutions Involving Negative Binomial Variates, 231	
12.6	Pascal-Poisson Distribution, 233	
12.7	A Riff-Shuffle Distribution, 234	

6 Hypergeometric Distributions	237
1 Definition, 237	
2 Historical Remarks and Genesis, 238	
2.1 Classical Hypergeometric Distribution, 238	
2.2 Negative (Inverse) Hypergeometric Distribution: Hypergeometric Waiting-Time Distribution, Beta-Binomial Distribution, 239	
2.3 Beta-Negative-Binomial Distribution: Beta-Pascal Distribution, Generalized Waring Distribution, 242	
2.4 Pólya Distributions: Generalized Hypergeometric Distributions, 244	
3 Moments, 249	
4 Properties, 253	
5 Approximations and Bounds, 256	
6 Tables and Computation, 262	
7 Estimation, 262	
7.1 Classical Hypergeometric Distribution, 262	
7.2 Negative (Inverse) Hypergeometric Distribution, 264	
7.3 Beta-Pascal Distribution, 266	
8 Characterizations, 266	
9 Applications, 269	
9.1 Classical Hypergeometric Distribution, 269	
9.2 Negative (Inverse) Hypergeometric Distribution: Beta-Binomial Distribution, 270	
9.3 Beta-Negative-Binomial Distribution: Beta-Pascal Distribution, Generalized Waring Distribution, 271	
10 Special Cases, 272	
10.1 Discrete Rectangular Distribution, 272	
10.2 Distribution of Leads in Coin Tossing, 274	
10.3 Yule Distribution, 275	
10.4 Waring Distribution, 278	
11 Extended Hypergeometric Distributions, 279	
12 Other Related Distributions, 282	
7 Logarithmic Distribution	285
1 Definition, 285	
2 Historical Remarks and Genesis, 286	
3 Moments, 288	
4 Properties, 290	

5	Approximations and Bounds,	291
6	Computation and Tables,	292
7	Estimation,	293
7.1	Model Selection,	293
7.2	Point Estimation and Confidence Intervals,	294
8	Characterizations,	297
9	Applications,	298
10	Truncated and Modified Logarithmic Distributions,	299
11	Other Related Distributions,	300
8	Mixture Distributions	305
1	Introduction,	305
2	Finite Mixtures of Discrete Distributions,	309
2.1	Parameters of Finite Mixtures,	309
2.2	Zero-Modified Distributions,	312
2.3	Finite Poisson Mixtures,	318
2.4	Finite Binomial Mixtures,	319
2.5	Other Finite Mixtures of Discrete Distributions,	321
3	Continuous and Countable Mixtures of Discrete Distributions,	322
3.1	Three Important Theorems,	322
3.2	Mixtures of Poisson Distributions,	326
3.3	Mixtures of Binomial Distributions,	335
3.4	Other Continuous and Countable Mixtures of Discrete Distributions,	337
9	Generalized (Stopped-Sum) Distributions	343
1	Introduction,	343
2	Damage Processes,	349
3	Poisson-Stopped-Sum Distributions: Generalized Poisson Distributions,	351
4	Hermite Distribution,	357
5	Poisson-Binomial Distribution,	364
6	Neyman Type A Distribution,	368
6.1	Definition,	368
6.2	Moment Properties,	371
6.3	Tables and Approximations,	372
6.4	Estimation,	374
6.5	Applications,	376
7	Pólya-Aeppli Distribution,	378

8	Poisson-Pascal Distribution: Poisson-Negative Binomial Distribution, Generalized Pólya-Aeppli Distribution,	382
9	Generalizations of the Neyman Type A Distribution,	386
10	Thomas Distribution,	392
11	Lagrangian Poisson Distribution: Shifted Borel-Tanner Distribution,	394
12	Other Families of Stopped-Sum Distributions,	400
10	Matching, Occupancy, and Runs Distributions	405
1	Introduction,	405
2	Probabilities of Combined Events,	406
3	Matching Distributions,	409
4	Occupancy Distributions,	414
4.1	Classical Occupancy and Coupon Collecting,	414
4.2	Maxwell-Boltzmann, Bose-Einstein, and Fermi-Dirac Statistics,	420
5	Runs Distributions,	422
5.1	Runs of Like Elements,	422
5.2	Runs Up and Down,	425
6	Distributions of Order k ,	426
6.1	Success Runs Distributions,	426
6.2	Philippou, Aki, and Hirano's Distributions of Order k ,	427
6.3	Further Distributions of Order k ,	432
11	Miscellaneous Discrete Distributions	433
1	Absorption Distribution,	433
2	Dandekar's Modified Binomial and Poisson Distributions,	435
3	Digamma and Trigamma Distributions,	436
4	Discrete Adès Distribution,	437
5	Discrete Student's t -Distribution,	438
6	Geeta Distribution,	439
7	Gegenbauer Distribution: Negative Binomial*Pseudo-Negative Binomial Convolution,	440
8	Gram-Charlier Type B Distributions,	442
9	"Interrupted" Distributions,	443
10	Lost-Games Distributions,	445
11	Naor's Distribution,	447
12	Partial-Sums Distributions,	448
13	Queueing Theory Distributions,	451

14	Record-Value Distributions,	453
15	Sichel Distribution: Poisson-Inverse Gaussian Distribution,	455
16	Skellam's Gene Frequency Distribution,	457
17	Steyn's Two-Parameter Power Series Distributions,	459
18	Univariate Multinomial-Type Distributions,	460
19	Urn Models with Stochastic Replacements,	463
20	Zipf and Zeta Distributions,	465
	Bibliography	473
	Abbreviations	549
	Index	551