

Contents

Preface	<i>page</i>	ix
1 Introduction		
1.1 Examples of stochastic processes		1
1.2 Specification of stochastic processes		13
1.3 Markov processes		16
Bibliographic notes and exercises		19
2 The Random Walk		
2.1 Introduction		22
2.2 The simple random walk		25
2.3 The general one-dimensional random walk in discrete time		46
2.4 Further topics		70
Bibliographic notes and exercises		73
3 Markov Chains		
3.1 Introduction		76
3.2 A two-state Markov chain		78
3.3 General definitions and some examples		84
3.4 The classification of states and the limit theorem		91
3.5 Closed sets of states		99
3.6 Irreducible chains and equilibrium distributions		101
3.7 Branching processes		102
3.8 Limiting properties of irreducible chains		106
3.9 Absorption problems		117
3.10 Non-negative square matrices		118
3.11 Finite Markov chains		123
3.12 Further topics		132
3.13 Appendix on power series with non-negative coefficients		139
Bibliographic notes and exercises		141
4 Markov Processes with Discrete States in Continuous Time		
4.1 The Poisson process		146
4.2 Generalizations of the Poisson process		153
4.3 Some simple processes of the birth-death type		156
4.4 Equilibrium distributions		171
4.5 General formulation		178
4.6 Some miscellaneous topics		186
Bibliographic notes and exercises		198

5	Markov Processes in Continuous Time with Continuous State Space	
5.1	Introduction	<i>page</i> 203
5.2	Continuous limit of the simple random walk: the Wiener process	205
5.3	The diffusion equations for the Wiener process	208
5.4	First passage problems for the Wiener process	210
5.5	Continuous limits of more general discrete processes	213
5.6	The Kolmogorov equations	215
5.7	Boundary conditions for homogeneous diffusion processes	219
5.8	The Ornstein–Uhlenbeck process	225
5.9	Transformations of the Wiener process	228
5.10	First passage times for homogeneous diffusion processes	230
5.11	Approximations to discrete processes by means of diffusion processes	234
5.12	Continuous and jump transitions	237
5.13	Processes with independent increments	243
5.14	Multidimensional processes	246
	Bibliographic notes and exercises	248
6	Non-Markovian Processes	
6.1	Introduction	252
6.2	The device of stages	257
6.3	Supplementary variables	262
6.4	Imbedded Markov process	266
	Bibliographic notes and exercises	269
7	Stationary Processes: Time Domain	
7.1	Introduction	272
7.2	Some definitions and special processes	275
7.3	Some general results about stationary processes	286
7.4	Processes in continuous time	293
7.5	Prediction theory	299
	Bibliographic notes and exercises	304
8	Stationary Processes: Frequency Domain	
8.1	Introduction	308
8.2	The spectral representation	313
8.3	Linear operations on stationary processes	319
8.4	Derivation of the spectral representation	323
8.5	Prediction and filtering theory	325
8.6	Multivariate processes	331
	Bibliographic notes and exercises	336

9 Point Processes

9.1	Introduction	<i>page</i>	338
9.2	The renewal process		340
9.3	Renewal processes with more than one type of interval		350
9.4	Stationary point processes		356
9.5	Operations on point processes		362
9.6	Real-valued processes associated with a point process		366
	Bibliographic notes and exercises		373
Appendix 1	Table of exponentially distributed random quantities		377
Appendix 2	Bibliography		378
	Author index		387
	Subject index		389