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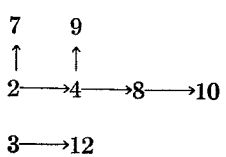
Each chapter leads most naturally to the next. Some chapters, however, lead fairly naturally to later ones, as this diagram indicates.