

# Contents

CHAPTER	PAGE
1 INTRODUCTION AND GENERAL ORIENTATION . . . . .	1
2 GENERATING FUNCTIONS . . . . .	5
2.1 Introduction . . . . .	5
2.2 Definitions and elementary results . . . . .	5
2.3 Convolutions . . . . .	7
2.4 Compound distributions . . . . .	9
2.5 Partial fraction expansions . . . . .	10
2.6 Moment- and cumulant-generating functions . . . . .	12
<i>Problems for solution</i> . . . . .	14
3 RECURRENT EVENTS . . . . .	16
3.1 Introduction . . . . .	16
3.2 Definitions . . . . .	16
3.3 Basic theorems . . . . .	18
3.4 Illustration . . . . .	19
3.5 Delayed recurrent events . . . . .	21
<i>Problems for solution</i> . . . . .	22
4 RANDOM WALK MODELS . . . . .	24
4.1 Introduction . . . . .	24
4.2 Gambler's ruin . . . . .	25
4.3 Probability distribution of ruin at $n$ th trial . . . . .	28
4.4 Extensions . . . . .	34
<i>Problems for solution</i> . . . . .	36
5 MARKOV CHAINS . . . . .	38
5.1 Introduction . . . . .	38
5.2 Notation and definitions . . . . .	39
5.3 Classification of states . . . . .	41
5.4 Classification of chains . . . . .	46
5.5 Evaluation of $P^n$ . . . . .	47
5.6 Illustrations . . . . .	51
<i>Problems for solution</i> . . . . .	56

6	DISCRETE BRANCHING PROCESSES . . . . .	58
6.1	Introduction . . . . .	58
6.2	Basic theory . . . . .	59
6.3	Illustration . . . . .	63
	<i>Problems for solution</i> . . . . .	64
7	MARKOV PROCESSES IN CONTINUOUS TIME . . . . .	66
7.1	Introduction . . . . .	66
7.2	The Poisson process . . . . .	67
7.3	Use of generating functions . . . . .	69
7.4	“Random-variable” technique . . . . .	70
7.5	Solution of linear partial differential equations . . . . .	74
7.6	General theory . . . . .	75
	<i>Problems for solution</i> . . . . .	82
8	HOMOGENEOUS BIRTH AND DEATH PROCESSES . . . . .	84
8.1	Introduction . . . . .	84
8.2	The simple birth process . . . . .	84
8.3	The general birth process . . . . .	88
8.4	Divergent birth processes . . . . .	89
8.5	The simple death process . . . . .	90
8.6	The simple birth-and-death process . . . . .	91
8.7	The effect of immigration . . . . .	97
8.8	The general birth-and-death process . . . . .	101
8.9	Multiplicative processes . . . . .	102
	<i>Problems for solution</i> . . . . .	105
9	SOME NON-HOMOGENEOUS PROCESSES . . . . .	107
9.1	Introduction . . . . .	107
9.2	The Pólya process . . . . .	107
9.3	A simple non-homogeneous birth-and-death process . . . . .	110
9.4	The effect of immigration . . . . .	115
	<i>Problems for solution</i> . . . . .	116
10	MULTI-DIMENSIONAL PROCESSES . . . . .	117
10.1	Introduction . . . . .	117
10.2	Population growth with two sexes . . . . .	119
10.3	The cumulative population . . . . .	120
10.4	Mutation in bacteria . . . . .	125
10.5	A multiple-phase birth process . . . . .	131
	<i>Problems for solution.</i> . . . . .	135
11	QUEUEING PROCESSES . . . . .	136
11.1	Introduction . . . . .	136
11.2	Equilibrium theory . . . . .	137
11.3	Queues with many servers . . . . .	143

11.4	Monte Carlo methods in appointment systems . . . . .	147
11.5	Non-equilibrium treatment of a simple queue . . . . .	149
11.6	First passage times . . . . .	157
	<i>Problems for solution</i> . . . . .	160
12	EPIDEMIC PROCESSES . . . . .	162
12.1	Introduction . . . . .	162
12.2	Simple epidemics . . . . .	164
12.3	General epidemics . . . . .	169
12.4	Recurrent epidemics . . . . .	177
12.5	Chain-binomial models . . . . .	182
	<i>Problems for solution</i> . . . . .	184
13	COMPETITION AND PREDATION . . . . .	186
13.1	Introduction . . . . .	186
13.2	Competition between two species . . . . .	187
13.3	A prey-predator model . . . . .	189
14	DIFFUSION PROCESSES . . . . .	194
14.1	Introduction . . . . .	194
14.2	Diffusion limit of a random walk . . . . .	195
14.3	Diffusion limit of a discrete branching process . . . . .	197
14.4	General theory . . . . .	199
14.5	Application to population growth . . . . .	205
15	APPROXIMATIONS TO STOCHASTIC PROCESSES . . . . .	207
15.1	Introduction . . . . .	207
15.2	Continuous approximations to discrete processes . . . . .	207
15.3	Saddle-point approximations . . . . .	214
15.4	Neglect of high-order cumulants . . . . .	217
15.5	Stochastic linearization . . . . .	219
16	SOME NON-MARKOVIAN PROCESSES . . . . .	222
16.1	Introduction . . . . .	222
16.2	Renewal theory and chromosome mapping . . . . .	224
16.3	Use of integral equations . . . . .	228
	REFERENCES . . . . .	234
	SOLUTIONS TO PROBLEMS . . . . .	237
	AUTHOR INDEX . . . . .	243
	SUBJECT INDEX . . . . .	245