

Contents

	Standard Notation	xv
Chapter 1	Stochastic Processes: Basic Concepts and Definitions	1
	1.1 Notation and Basic Definitions	1
	1.2 Probability Measures Associated with Stochastic Processes	6
Chapter 2	Martingales and the Wiener Process	12
	2.1 The Wiener Process	12
	2.2 Martingales and Supermartingales	18
	2.3 Properties of Wiener Processes—Wiener Martingales	20
	2.4 Decomposition of Supermartingales	24
	2.5 The Quadratic Variation of a Square Integrable Martingale	33
	2.6 Local Martingales	37
	2.7 Some Useful Theorems	45
Chapter 3	Stochastic Integrals	48
	3.1 Predictable Processes	48
	3.2 Stochastic Integrals for L^2 -Martingales	52
	3.3 The Ito Integral	59
	3.4 The Stochastic Integral with Respect to Continuous Local Martingales	70
Chapter 4	The Ito Formula	77
	4.1 Vector-Valued Processes	77
	4.2 The Ito Formula	78

	4.3 Ito Formula (General Version)	84
	4.4 Applications of the Ito Formula	88
	4.5 A Vector-Valued Version of Ito's Formula	92
Chapter 5	Stochastic Differential Equations	94
	5.1 Existence and Uniqueness of Solutions	94
	5.2 Strong and Weak Solutions	105
	5.3 Linear Stochastic Differential Equations	108
	5.4 Markov Processes	111
	5.5 Extended Generator of $\xi(t)$	120
	5.6 Diffusion Processes	124
	5.7 Existence of Moments	127
Chapter 6	Functionals of a Wiener Process	134
	6.1 Introduction	134
	6.2 The Multiple Wiener Integral	134
	6.3 Hilbert Spaces Associated with a Gaussian Process	139
	6.4 Tensor Products and Symmetric Tensor Products of Hilbert Spaces	139
	6.5 CONS in $\sigma[\otimes^p H(R)]$ and $\sigma[\otimes^p L_1(X)]$	144
	6.6 Homogeneous Chaos	145
	6.7 Stochastic (Ito) Integral Representation	155
	6.8 A Generalization of Theorem 6.7.3	159
Chapter 7	Absolute Continuity of Measures and Radon-Nikodym Derivatives	162
	7.1 Exponential Supermartingales, Martingales, and Girsanov's Theorem	162
	7.2 Sufficient Conditions for the Validity of Girsanov's Theorem	172
	7.3 Stochastic Equations and Absolute Continuity of Induced Measures	174
	7.4 Weak Solutions	179
	7.5 Stochastic Equations Involving Vector-Valued Processes	181
	7.6 Explosion Times and an Extension of Girsanov's Formula	182
	7.7 Nonexistence of a Strong Solution	189
Chapter 8	The General Filtering Problem and the Stochastic Equation of the Optimal Filter (Part I)	192
	8.1 The Filtering Problem and the Innovation Process	192
	8.2 Observation Process Model with Absolutely Continuous (S_t)	204

8.3	Stochastic Integral Representation of a Separable Martingale on $(\Omega, \mathcal{F}_t, P)$	208
8.4	A Stochastic Equation for the General Nonlinear Filtering Problem	210
8.5	Applications	220
8.6	The Case of Markov Processes	221
Chapter 9	Gaussian Solutions of Stochastic Equations	225
9.1	The Gohberg-Krein Factorization Theorem	225
9.2	Nonanticipative Representations of Equivalent Gaussian Processes	230
9.3	Nonanticipative Representation of a Gaussian Process Equivalent to a Wiener Process	232
9.4	Gaussian Solutions of Stochastic Equations	233
9.5	Vector-Valued Processes	244
Chapter 10	Linear Filtering Theory	247
10.1	Introduction	247
10.2	The Stochastic Model for the Kalman Theory	252
10.3	Derivation of the Kalman Filter from the Nonlinear Theory	256
10.4	The Filtering Problem for Gaussian Processes	260
10.5	The Kalman Filter (Independent Derivation)	266
Chapter 11	The Stochastic Equation of the Optimal Filter (Part II)	273
11.1	Introduction	273
11.2	A Stochastic Differential Equation for the Conditional Density	274
11.3	A Bayes Formula for Stochastic Processes	278
11.4	Equality of the Sigma Fields \mathcal{F}_t^z and \mathcal{F}_t^y	283
11.5	Solution of the Filter Equation	287
	Notes	295
	References	305
	Index of Commonly Used Symbols	311
	Index	313