Contents

Acknowledgments xi

Introduction xiii

Special Notations xix

I Martingales 1

1. Histories and Stopping Times 1
2. Martingales 3
3. Predictability 8
4. Square-Integrable Martingales 11
References 12
Solutions to Exercises, Chapter I 13

II Point Processes, Queues, and Intensities 18

1. Counting Processes and Queues 18
2. Watanabe's Characterization 23
3. Stochastic Intensity, General Case 27
4. Predictable Intensities 30
5. Representation of Queues 35
6. Random Changes of Time 40
7. Cryptographic Point Processes 43
References 47
Solutions to Exercises, Chapter II 48

III Integral Representation of Point-Process Martingales 56

1. The Structure of Internal Histories 56
2. Regenerative Form of the Intensity 59
3. The Representation Theorem 64
4. Hilbert-Space Theory of Poissonian Martingales 70
5. Useful Extensions 75
References 76
Solutions to Exercises, Chapter III 77

IV Filtering 83

1. The Theory of Innovations 83
2. State Estimates for Queues and Markov Chains 100
3. Continuous States and Nontrivial Prehistory 107
References 115
Solutions to Exercises, Chapter IV 115

V Flows in Markovian Networks of Queues 122

1. Single Station: The Historical Results and the Filtering Method 122
2. Jackson's Networks 131
3. Burke's Output Theorem for Networks 138
4. Cascades and Loops in Jackson's Networks 143
5. Independence and Poissonian Flows in Markov Chains 151
References 154
Solutions to Exercises, Chapter V 155

VI Likelihood Ratios 158

1. Radon–Nikodym Derivatives and Tests of Hypotheses 158
2. Changes of Intensities "à la Girsanov" 165
3. Filtering by the Method of the Probability of Reference 170
4. Applications 174
5. The Capacity of a Point-Process Channel 180
6. Detection Formula 187
References 189
Solutions to Exercises, Chapter VI 190

VII Optimal Control 196

1. Modeling Intensity Controls 196
2. Dynamic Programming for Intensity Controls: Complete-Observation Case 202
3. Input Regulation. A Case Study in Impulsive Control 211
4. Attraction Controls 219
5. Existence via Likelihood Ratio 225
References 229
Solutions to Exercises, Chapter VII 230

VIII Marked Point Processes 233

1. Counting Measure and Intensity Kernels 233
2. Martingale Representation and Filtering 238
3. Radon–Nikodym Derivatives 241
4. Towards a General Theory of Intensity 244
References 250
Solutions to Exercises, Chapter VIII 250

A1 Background in Probability and Stochastic Processes 255

1. Introduction 255
2. Monotone Class Theorem 256
3. Random Variables 261
4. Expectations 266
5. Conditioning and Independence 275
6. Convergence 285
7. Stochastic Processes 287
8. Markov Processes 290
References 295

A2 Stopping Times and Point-Process Histories 296

1. Stopping Times 296
2. Changes of Time and Meyer–Dellacherie's Integration Formula 300
3. Point-Process Histories 303
References 311

A3 Wiener-Driven Dynamical Systems 312

1. Ito's Stochastic Integral 312
2. Square-Integrable Brownian Martingales 321
3. Girsanov's Theorem 327
References 332

A4 Stieltjes–Lebesgue Calculus 334

1. The Stieltjes–Lebesgue Integral 334
2. The Product and Exponential Formulas 336
References 339

General Bibliography 341

Index 351