CONTENTS

PREFACE xi

Acknowledgments, xii

1 **INTRODUCTION** 1

Organization of the Book, 4

2 **PRELIMINARIES: SET THEORY, MAPPINGS, AND LINEAR SYSTEMS** 8

Set Theory, 8

Examples of proofs, 15

Mappings and Functions, 18

Linear System Fundamentals, 19

Exercises, 23

3 **PROBABILITY SPACES** 27

Sample Spaces, 33

Examples of sample spaces, 33

Event Spaces, 36

Examples of event spaces, 40

Probability Measures, 47

Examples of probability spaces, 50

Exercises, 59

4 RANDOM VARIABLES, VECTORS, AND PROCESSES 64

Examples of Random Variables, 65
Random Variables, 70
Distributions of Random Variables, 75
Examples of Derived Distributions, 81
Random Vectors, 84
Marginal and Joint Distributions, 88
Examples of Random Vectors, 94
Independence, 95
Elementary Conditional Probability 98,
Random Processes, 99
Simple Examples of Random Processes, 101
Exercises, 105

5 SPECIFICATION OF RANDOM PROCESSES 111

Process Distributions, 112
Examples of Specification, 120
Exercises, 123

6 EXPECTATION AND LAWS OF LARGE NUMBERS 126

Expectation, 130
Examples, 131
Expectations of functions of random variables, 133
Examples, 134
Functions of Several Random Variables, 136
Properties of Expectation, 137
Correlation, 139
Sample Averages, 143
Convergence and Laws of Large Numbers, 145
Convergence with probability one vs. convergence in probability, 150
Convergence in Distribution, 151
Exercises, 152

7 STATIONARITY AND ERGODIC PROPERTIES 162

Stationarity, 163
Examples of Stationary Processes, 166
I.i.d. processes, 166
Gaussian processes, 167

The Shift, 168
Ergodic Theorems, 170
Two Mean Ergodic Theorems, 171
Ergodicity, 174
Exercises, 178

8 SECOND-ORDER MOMENTS AND LINEAR SYSTEMS 181

Linear Filtering of Random Processes, 182
Discrete Time Second-Order I/O Relations, 184
Linearly Filtered Uncorrelated Processes, 186
Continuous Time Linear Filters, 189
Linear Modulation, 190
Power Spectral Densities, 192
White Noise, 197
Differentiating Random Processes, 200
Linear Estimation, 202
Exercises, 215

9 USEFUL RANDOM PROCESSES 224

Discrete Time Linear Models, 225
Processes with Independent and Stationary Increments, 229
A Counting Process, 233
Derived Distributions of Sums of Independent Random Variables, 235
The Binomial Counting Process, 237
A Discrete Random Walk, 238
The Discrete Time Wiener Process, 239
Conditional pmf's and Specification, 240
Specification of Discrete Independent Increment Processes, 242
Conditional pdf's and Specification, 244
Conditional Probability, 245
Specification of the Discrete Time Wiener Process, 248
Second-order Moments of Continuous Time I.s.i. Processes, 249
Specification of Continuous Time I.s.i. Processes, 251
Moving-average and Autoregressive Processes, 254
The Discrete Time Gauss-Markov Process, 255

Gaussian Random Processes, 256
Exponential Modulation, 258
Exercises, 262

10 COMPOUND PROCESSES AND CONDITIONAL EXPECTATION **268**
Discrete Time Compound Processes, 269
Conditional Expectation, 271
An Application to Estimation Theory, 273
Continuous Time Compound Processes, 274
Exercises, 275

11 MODELING PHYSICAL PROCESSES **277**
The Central Limit Theorem, 278
The Poisson Counting Process, 282
Thermal Noise, 285
Exercises, 287

APPENDIX: SUPPLEMENTARY READING **290**
References, 294

INDEX **297**