
Contents

Preface ix

Introduction 1

1 Background material 7

- 1.1 Abstract Wiener spaces 7
- 1.2 Stochastic integration 10
- 1.3 Preliminary results 13

2 The functional analytic approach 16

- 2.1 Symmetric diffusion operators 16
- 2.2 A symmetric diffusion operator for Wiener space 20
- 2.3 Regularity of measures induced by Wiener functionals 26
- 2.4 Application of the Malliavin calculus to stochastic differential equations 28

3 The variational approach 34

- 3.1 Perturbation via the Girsanov theorem 34
- 3.2 Smooth dependence of stochastic differential equations upon parameters 37
- 3.3 Regularity of the measures induced by a stochastic differential equation 41

4 An elementary derivation of Malliavin's inequalities 46

- 4.1 Introduction 46
- 4.2 A sequence of differentiable approximations to the Itô map 48
- 4.3 Derivation of the covariance matrix 53
- 4.4 Regularity of the measures induced by a stochastic differential equation 56

- 5 A discussion of the different forms of the theory 63**
- 5.1 An outline of Malliavin's original paper 63
 - 5.2 A condition for equivalence of the approaches of Stroock and Bismut 65
 - 5.3 Transformation theorems and integration by parts operators 68
- 6 Non-degeneracy of the covariance matrix under Hörmander's condition 73**
- 6.1 Hörmander's theorem 73
 - 6.2 Hörmander's condition implies $\sigma^{-1} \in L^p$ for all p 77
- 7 Some further applications of the Malliavin calculus 87**
- 7.1 The filtering problem 87
 - 7.2 A study of an infinite system of interacting particles 92
 - 7.3 Towards the construction of quasi-invariant measures on an infinite dimensional vector space 97
- References 103**
- Index 105**