

Contents

1. Elements of Probability Theory	1
1.1 Probability and Probability Spaces	1
1.1.1 Measurable Spaces, Measurable Mappings and Measure Spaces	1
1.1.2 Probability Spaces	3
1.2 Random Variables and “Almost Sure” Properties	3
1.2.1 Mathematical Expectations	5
1.2.2 Probability Distribution and Density Functions	7
1.2.3 Characteristic Function	9
1.2.4 Examples	9
1.3 Random Vectors	11
1.3.1 Stochastic Independence	13
1.3.2 The Gaussian N Vector and Gaussian Manifolds	16
1.4 Stochastic Processes	18
1.4.1 The Hilbert Space $L_2(\Omega)$	21
1.4.2 Second-Order Processes	24
1.4.3 The Gaussian Process	26
1.4.4 Brownian Motion, the Wiener-Lévy Process and White Noise	27
2. Calculus in Mean Square	30
2.1 Convergence in Mean Square	30
2.2 Continuity in Mean Square	33
2.3 Differentiability in Mean Square	35
2.3.1 Supplementary Exercises	39
2.4 Integration in Mean Square	40
2.4.1 Some Elementary Properties	42
2.4.2 A Condition for Existence	46
2.4.3 A Strong Condition for Existence	52
2.4.4 A Weak Condition for Existence	54
2.4.5 Supplementary Exercises	63
2.5 Mean-Square Calculus of Random N Vectors	65
2.5.1 Conditions for Existence	67

2.6	The Wiener-Lévy Process	68
2.6.1	The General Wiener-Lévy N Vector	70
2.6.2	Supplementary Exercises	75
2.7	Mean-Square Calculus and Gaussian Distributions	76
2.8	Mean-Square Calculus and Sample Calculus	76
2.8.1	Supplementary Exercise	79
3.	The Stochastic Dynamic System	80
3.1	System Description	80
3.2	Uniqueness and Existence of m.s. Solution to (3.3)	81
3.2.1	The Banach Space $L_2^N(\Omega)$	81
3.2.2	Uniqueness	83
3.2.3	The Homogeneous System	84
3.2.4	The Inhomogeneous System	89
3.2.5	Supplementary Exercises	93
3.3	A Discussion of System Representation	94
4.	The Kalman-Bucy Filter	100
4.1	Some Preliminaries	101
4.1.1	Supplementary Exercise	102
4.2	Some Aspects of $L_2([a, b])$	103
4.2.1	Supplementary Exercise	106
4.3	Mean-Square Integrals Continued	106
4.4	Least-Squares Approximation in Euclidean Space	114
4.4.1	Supplementary Exercises	116
4.5	A Representation of Elements of $H(\mathbf{Z}, t)$	116
4.5.1	Supplementary Exercises	128
4.6	The Wiener-Hopf Equation	128
4.6.1	The Integral Equation (4.106)	134
4.7	Kalman-Bucy Filter and the Riccati Equation	146
4.7.1	Recursion Formula and the Riccati Equation	148
4.7.2	Supplementary Exercise	153
5.	A Theorem by Liptser and Shiriyayev	154
5.1	Discussion on Observation Noise	154
5.2	A Theorem of Liptser and Shiriyayev	155
	Appendix: Solutions to Selected Exercises	158
	References	167
	Subject Index	169