

Contents

Introduction	1
Chapter I	
Preliminaries	3
1. Basic Definitions and Notation	3
2. Martingales	8
3. The Poisson Process and Brownian Motion	13
4. Lévy Processes	20
5. Local Martingales	33
6. Stieltjes Integration and Change of Variables	35
7. Naive Stochastic Integration Is Impossible	40
Bibliographic Notes	41
Chapter II	
Semimartingales and Stochastic Integrals	43
1. Introduction to Semimartingales	43
2. Stability Properties of Semimartingales	44
3. Elementary Examples of Semimartingales	47
4. Stochastic Integrals	48
5. Properties of Stochastic Integrals	52
6. The Quadratic Variation of a Semimartingale	58
7. Itô's Formula (Change of Variables)	70
8. Applications of Itô's Formula	77
Bibliographic Notes	85
Chapter III	
Semimartingales and Decomposable Processes	87
1. Introduction	87
2. The Doob-Meyer Decompositions	90

3. Quasimartingales	95
4. The Fundamental Theorem of Local Martingales	98
5. Classical Semimartingales	105
6. Girsanov's Theorem	108
7. The Bichteler-Dellacherie Theorem	114
8. Natural Versus Predictable Processes	117
Bibliographic Notes	122
Chapter IV	
General Stochastic Integration and Local Times	123
1. Introduction	123
2. Stochastic Integration for Predictable Integrands	123
3. Martingale Representation	147
4. Stochastic Integration Depending on a Parameter	157
5. Local Times	162
6. Azema's Martingale	180
Bibliographic Notes	185
Chapter V	
Stochastic Differential Equations	187
1. Introduction	187
2. The H^p Norms for Semimartingales	188
3. Existence and Uniqueness of Solutions	193
4. Stability of Stochastic Differential Equations	201
5. Fisk-Stratonovich Integrals and Differential Equations	215
6. The Markov Nature of Solutions	235
7. Flows of Stochastic Differential Equations: Continuity and Differentiability	245
8. Flows as Diffeomorphisms: The Continuous Case	255
9. General Stochastic Exponentials and Linear Equations	266
10. Flows as Diffeomorphisms: The General Case	272
Bibliographic Notes	283
References	285
Symbol Index	295
Subject Index	297