

# Contents

<b>Preface</b>	<b>xi</b>
<b>Sample Course Outline</b>	<b>xiii</b>
<b>I Random Walk and Brownian Motion</b>	<b>1</b>
1. What is a Stochastic Process?, 1	
2. The Simple Random Walk, 3	
3. Transience and Recurrence Properties of the Simple Random Walk, 5	
4. First Passage Times for the Simple Random Walk, 8	
5. Multidimensional Random Walks, 11	
6. Canonical Construction of Stochastic Processes, 15	
7. Brownian Motion, 17	
8. The Functional Central Limit Theorem (FCLT), 20	
9. Recurrence Probabilities for Brownian Motion, 24	
10. First Passage Time Distributions for Brownian Motion, 27	
11. The Arcsine Law, 32	
12. The Brownian Bridge, 35	
13. Stopping Times and Martingales, 39	
14. Chapter Application: Fluctuations of Random Walks with Slow Trends and the Hurst Phenomenon, 53	
Exercises, 62	
Theoretical Complements, 90	
<b>II Discrete-Parameter Markov Chains</b>	<b>109</b>
1. Markov Dependence, 109	
2. Transition Probabilities and the Probability Space, 110	

3. Some Examples, 113
4. Stopping Times and the Strong Markov Property, 117
5. A Classification of States of a Markov Chain, 120
6. Convergence to Steady State for Irreducible and Aperiodic Markov Processes on Finite Spaces, 126
7. Steady-State Distributions for General Finite-State Markov Processes, 132
8. Markov Chains: Transience and Recurrence Properties, 135
9. The Law of Large Numbers and Invariant Distributions for Markov Chains, 138
10. The Central Limit Theorem for Markov Chains, 148
11. Absorption Probabilities, 151
12. One-Dimensional Nearest-Neighbor Gibbs States, 162
13. A Markovian Approach to Linear Time Series Models, 166
14. Markov Processes Generated by Iterations of I.I.D. Maps, 174
15. Chapter Application: Data Compression and Entropy, 184  
Exercises, 189  
Theoretical Complements, 214

### III Birth–Death Markov Chains

233

1. Introduction to Birth–Death Chains, 233
2. Transience and Recurrence Properties, 234
3. Invariant Distributions for Birth–Death Chains, 238
4. Calculations of Transition Probabilities by Spectral Methods, 241
5. Chapter Application: The Ehrenfest Model of Heat Exchange, 246  
Exercises, 252  
Theoretical Complements, 256

### IV Continuous-Parameter Markov Chains

261

1. Introduction to Continuous-Time Markov Chains, 261
2. Kolmogorov's Backward and Forward Equations, 263
3. Solutions to Kolmogorov's Equations in Exponential Form, 267
4. Solutions to Kolmogorov's Equations by Successive Approximation, 271
5. Sample Path Analysis and the Strong Markov Property, 275
6. The Minimal Process and Explosion, 288
7. Some Examples, 292
8. Asymptotic Behavior of Continuous-Time Markov Chains, 303
9. Calculation of Transition Probabilities by Spectral Methods, 314
10. Absorption Probabilities, 318

11. Chapter Application: An Interacting System: The Simple Symmetric Voter Model, 324  
 Exercises, 333  
 Theoretical Complements, 349
- V Brownian Motion and Diffusions 367**
1. Introduction and Definition, 367
  2. Kolmogorov's Backward and Forward Equations, Martingales, 371
  3. Transformation of the Generator under Relabeling of the State Space, 381
  4. Diffusions as Limits of Birth–Death Chains, 386
  5. Transition Probabilities from the Kolmogorov Equations: Examples, 389
  6. Diffusions with Reflecting Boundaries, 393
  7. Diffusions with Absorbing Boundaries, 402
  8. Calculation of Transition Probabilities by Spectral Methods, 408
  9. Transience and Recurrence of Diffusions, 414
  10. Null and Positive Recurrence of Diffusions, 420
  11. Stopping Times and the Strong Markov Property, 423
  12. Invariant Distributions and the Strong Law of Large Numbers, 432
  13. The Central Limit Theorem for Diffusions, 438
  14. Introduction to Multidimensional Brownian Motion and Diffusions, 441
  15. Multidimensional Diffusions under Absorbing Boundary Conditions and Criteria for Transience and Recurrence, 448
  16. Reflecting Boundary Conditions for Multidimensional Diffusions, 460
  17. Chapter Application: G. I. Taylor's Theory of Solute Transport in a Capillary, 468  
 Exercises, 475  
 Theoretical Complements, 497
- VI Dynamic Programming and Stochastic Optimization 519**
1. Finite-Horizon Optimization, 519
  2. The Infinite-Horizon Problem, 525
  3. Optimal Control of Diffusions, 533
  4. Optimal Stopping and the Secretary Problem, 542
  5. Chapter Application: Optimality of  $(S, s)$  Policies in Inventory Problems, 549  
 Exercises, 557  
 Theoretical Complements, 559

<b>VII An Introduction to Stochastic Differential Equations</b>	<b>563</b>
1. The Stochastic Integral, 563	
2. Construction of Diffusions as Solutions of Stochastic Differential Equations, 571	
3. Itô's Lemma, 582	
4. Chapter Application: Asymptotics of Singular Diffusions, 591	
Exercises, 598	
Theoretical Complements, 607	
<b>0 A Probability and Measure Theory Overview</b>	<b>625</b>
1. Probability Spaces, 625	
2. Random Variables and Integration, 627	
3. Limits and Integration, 631	
4. Product Measures and Independence, Radon–Nikodym Theorem and Conditional Probability, 636	
5. Convergence in Distribution in Finite Dimensions, 643	
6. Classical Laws of Large Numbers, 646	
7. Classical Central Limit Theorems, 649	
8. Fourier Series and the Fourier Transform, 653	
<b>Author Index</b>	<b>665</b>
<b>Subject Index</b>	<b>667</b>