

Contents

Introduction	vii
Chapter 1 – Aspects of the Ising Model.	1
1. Introduction.....	1
2. Peierls's Argument.....	4
3. Imposed Field, Thermodynamic Quantities.....	8
4. The Unidimensional Case and Tree.....	12
5. Antiferromagnetism.....	17
Chapter 2 – Gibbs Measures.....	19
1. The DLR Problem.....	19
2. The Structure of \mathcal{G}	26
3. Gibbs Specifications, Gibbs Measures.....	31
4. Relations with Thermodynamics.....	37
Chapter 3 – The Existence of Gibbs Measures.....	40
1. Markovian Properties.....	40
2. Existence of Gibbs Measures when Φ is Finite.....	43
3. Existence of Gibbs Measures – the General Case.....	45
4. Models of the $P(\varphi)$ -Type, Gaussian Gibbs Measures	53
Chapter 4 – Phase Transitions – 1: Methods of Convex Analysis.....	58
1. Holley's Inequality.....	60
2. The FKG Inequality.....	63
3. Attracting Specifications	65
4. The Ising Model on \mathbb{Z}^2	68
5. Symmetry Breaking on \mathbb{Z}^d for $d \geq 3$	77
Chapter 5 – Other Inequalities.....	82
1. Ursell Functions.....	82
2. GKS Inequalities	83
3. The GHS Inequality.....	90
4. The Simon-Lieb Inequality.....	92
5. Additional Inequalities.....	97

Chapter 6 — Phase Transitions — 2 : Phase Diagrams and Perturbed Hamiltonians	99
1. Fundamental Configurations	99
2. Perturbed Hamiltonians	102
3. The Pirogov-Sinai Theorem	110
4. Boundary Models	112
5. Principle of the Proof	118
Chapter 7 — Phase Transitions — 3: Positive Reflexivity	120
1. Reflexive Positivity	120
2. The Unidimensional Case	123
3. Checkerboard Estimate	127
4. Gaussian Domination	131
5. The Infrared Domination	132
Chapter 8 — Continuous Symmetry and Other Methods	136
1. The Case of Continuous Symmetry	136
2. The Lack of Symmetry Breaking when $d = 2$	139
3. Spontaneous Magnetization when $d \geq 3$	145
4. Onsager's Method	146
5. Combinatorial Approach	154
6. The Lee-Yang Theorem	157
Chapter 9 — The Dynamics of Ising Systems	161
1. Introduction	161
2. A Finite Number of Sites	164
3. An Infinite Number of Sites	169
Chapter 10 — Statistics and Applications	174
1. Ergodicity	175
2. Statistics	181
3. Image Processing	188
4. Other Applications	201
Bibliography	205
Index	217