CONTENTS

PREFACE ix

CONTENTS xv

STANDARD NOTATION xvii

CHAPTER 1 EXAMPLES AND AUXILIARY RESULTS 1

1.0. Introduction 1
1.1. Examples of Stochastic Evolution Systems 1
1.2. Measurability and Integrability in Banach Spaces 5
1.3. Martingales in $\mathbf{R}^1$ 10
1.4. Diffusion Processes 20

CHAPTER 2 STOCHASTIC INTEGRATION IN A HILBERT SPACE 39

2.0. Introduction 39
2.1. Martingales and Local Martingales 39
2.2. Stochastic Integrals with Respect to Square Integrable Martingale 52
2.3. Stochastic Integrable with Respect to a Local Martingale 63
2.4. An Energy Equality in a Rigged Hilbert Space 70

CHAPTER 3 LINEAR STOCHASTIC EVOLUTION SYSTEMS IN HILBERT SPACES 86

3.0. Introduction 86
3.1. Coercive Systems 87
3.2. Dissipative Systems 102
3.3. Uniqueness and the Markov Property 108
3.4. The First Boundary Problem for Ito's Partial Differential Equations 114

CHAPTER 4 ITO'S SECOND ORDER PARABOLIC EQUATIONS 125

4.0. Introduction 125
4.1. The Cauchy Problem for Superparabolic Ito's Second Order Parabolic Equations 129
4.2. The Cauchy Problem for Ito's Second Order Equations 154
4.3. The Forward Cauchy Problem and the Backward One in Weighted Sobolev Spaces 164

CHAPTER 5 ITO'S PARTIAL DIFFERENTIAL EQUATIONS AND DIFFUSION PROCESSES 175

5.0. Introduction 175
5.1. The Method of Stochastic Characteristics 179
5.2. Inverse Diffusion Processes, the Method of Variation of Constants and the Liouville Equations 193
5.3. A Representation of a Density-valued Solution 202

CHAPTER 6 FILTERING INTERPOLATION AND EXTRAPOLATION OF DIFFUSION PROCESSES 221

6.0. Introduction 221
6.1. Bayes' Formula and the Conditional Markov Property 223
6.2. The Forward Filtering Equation 229
6.3. The Backward Filtering Equation Interpolation and Extrapolation 239

CHAPTER 7 HYPOELLIPTICITY OF ITO'S SECOND ORDER PARABOLIC EQUATIONS 251

7.0. Introduction 251
7.1. Measure-valued Solution and Hypoellipticity under Generalized Hörmander's Condition 252
7.2. The Filtering Transition Density and a Fundamental Solution of the Filtering Equation in Hypoelliptic and Superparabolic Cases 280

NOTES 294

REFERENCES 301

INDEX 311