

CONTENTS

Preface	x
Review	1
Linear Algebra	1
Rectangular Matrices	1
Square Matrices	3
Matrices as Linear Transformation	4
Sequences of Matrices	9
Probability	10
Distributions; Characteristic Functions	10
Vector/Multidimensional Random Variables	15
Functions of Random Variables	17
Convergence of Random Variables	21
Fourier Series	22
Fourier Transforms	25
Some Frequently Used Inequalities	28
Almost Periodic Sequences	29
Gradient	30
Problems	31
Notes and Comments	34
References	34
1 Random Processes: Basic Concepts, Properties	37
1.1 Random Processes: Basic Definitions, Concepts	37
1.1.1 Examples: Continuous-Parameter Processes	38
1.1.2 Examples: Discrete-Parameter Processes	42
1.1.3 Sample Paths: Continuous-Parameter Processes	44
1.1.4 Sample Paths: Discrete-Parameter Processes	46
1.1.5 Vector-Valued Processes	47

Problems	48
1.2 Moments of Random Processes	50
1.2.1 Continuous-Parameter Processes: Continuity in the Mean Square	53
1.2.2 Examples	55
1.2.3 Equivalent Gaussian Processes	57
Problems	58
Notes and Comments	61
References	62
2 Stationary Random Processes: Covariance and Spectrum	63
2.1 Constructing Covariance Functions: Continuous-Parameter Processes	65
2.1.1 Covariance Functions: Examples	65
2.2 Constructing Covariance Functions: Discrete-Parameter Processes	74
2.3 Characterization of Covariance Functions: Spectral Representation Theorem	78
2.3.1 Continuous-Parameter Processes	78
2.3.2 Discrete-Parameter Processes	84
2.4 Elementary Properties of Stationary Covariance Functions: Continuous-Time Models	89
Problems	95
Notes and Comments	107
References	107
3 Response of Linear Systems to Random Inputs: Discrete-Time Models	109
3.1 "Weighting Pattern" System Models	109
3.1.1 Systems with Finite Memory	110
3.1.2 Physically Nonrealizable Systems	113
3.2 State-Space Models	114
3.2.1 Steady-State (or Asymptotic) Solution	117
3.2.2 Steady-State Spectral Density	126
3.3 Steady-State Noise Response: General Cases	131
3.3.1 State-Space Systems Models and Kalman Signal Generation Models	131
3.3.2 Steady-State Response: Weighting Pattern Models	134
3.3.3 Weighting Pattern Signal Generation Model	138
3.3.4 "Prewhitening" Filters	144

3.4	Difference Equation and ARMA Models	149
3.5	Time-Varying Systems	154
	Problems	154
	Notes and Comments	161
	References	161
4	Response of Linear Systems to Random Inputs: Continuous-Time Models	163
4.1	Definition of Integrals with Stochastic Integrands	163
4.2	Steady-State Response: Weighting Function Models	169
4.3	White Noise and Signal Generation Models	173
4.3.1	White Noise: Continuous Parameter	173
4.3.2	Steady-State Response to White Noise Inputs	178
4.3.3	Weighting Function Signal Generation Model	180
4.4	State-Space Models	181
4.4.1	Steady-State Solution	184
4.5	Examples	193
4.6	Derivatives of Stochastic Processes	198
4.7	Continuity of Sample Paths	202
4.8	Example: Noise in Electric Circuits	203
	Problems	204
	Notes and Comments	213
	References	213
5	Time Averages and the Ergodic Principle	215
5.1	Discrete-Time Models	215
5.1.1	One-Dimensional Example	217
5.1.2	Estimate of Covariance	219
5.1.3	More General Statistics	221
5.1.4	Failure of Ergodic Principle	225
5.1.5	Spectral Density Estimates	228
5.1.6	Example: Delta Function Spectral Density	233
5.1.7	Signal Generation Models	237
5.2	Continuous-Time Models	239
5.2.1	Signal Generation Models	240
	Problems	245
	Notes and Comments	247
	References	248

6 Sampling Principle and Interpolation	249
6.1 Karhunen–Loeve Expansion	261
6.2 Discrete–Time Models	263
6.3 Discrete–Time Approximation	268
Problems	269
Notes	273
References	273
7 Simulation of Random Processes	275
7.1 IID Sequences: White Noise	275
7.2 Signal-Generation Models	277
7.2.1 Rice Model	277
7.2.2. Kalman (State–Space) Models	282
7.2.3 Weighting Pattern Models	282
7.3 Continuous–Time Processes	287
7.3.1 Approximation	289
7.3.2 Simulation	294
Problems	295
Notes and Comments	295
References	295
8 Random Fields	297
8.1 Random Fields in Three-Space	297
8.1.1 Homogeneous Fields	298
8.2 Random Fields in Two-Space	312
8.2.1 Isotropic Fields	312
8.2.2 Sampling Principle	315
8.2.3 Response of Linear Systems	317
8.3 Fields on Lattices (Grids)	318
8.3.1 Homogeneous Fields	318
8.3.2 Simulation of Fields on Two-Dimensional Grids	322
Problems	328
Notes and Comments	333
References	334
9 Linear Filtering Theory	335
9.1 Best Linear Estimate	335
9.1.1 Minimizing the Error Covariance Matrix	338
9.1.2 Extension to Continuous–Time Processes	339

9.2	Conditional Expectation	343
9.2.1	Variational Definition of Conditional Expectation	346
9.2.2	Conditional Expectation: The Gaussian Case	347
9.2.3	Conditional Density: The Gaussian Case	349
9.2.4	Mutual Information	352
9.3	Gram-Schmidt Orthogonalization and Covariance Matrix Factorization	353
9.3.1	Simulating Gaussian Vectors	356
9.4	The Maximum Likelihood Principle	358
9.5	Estimation of Signal Parameters in Additive Noise Models	359
9.5.1	The “Bayesian” Model	361
9.5.2	Conditional Expectation Estimate	361
9.5.3	Maximum Likelihood Estimate	362
9.5.4	Fit Error	365
9.6	Example: Polynomial in Noise: Curve Fitting	366
9.6.1	Maximum Likelihood Estimate: Maximum Ignorance	368
9.7	A Tracking Problem: Linear Model	369
9.7.1	Miscellaneous Ad Hoc Estimates	372
9.7.2	Time-Varying Model	373
9.7.3	On-Line (“Recursive”) Estimation	373
9.7.4	Example: System Identification	375
9.8	Linear Filtering: General Case	380
9.8.1	Stationary Processes: Steady-State Response	380
9.8.2	Martingales	381
9.8.3	Linear Smoothing: Interpolation	384
9.8.4	Example: Signal in Additive White Noise	386
9.8.5	Wiener Filters/Kalman Filters	387
	Problems	389
	Notes and Comments	395
	References	395
	Index	397