

Contents

<i>Preface</i>	<i>xi</i>
<i>Acknowledgments</i>	<i>xiii</i>
<i>List of Symbols</i>	<i>xv</i>

I Introduction

I.1 Introduction	1
I.2 Prerequisites	2
I.3 Scope and Objectives	3
I.4 Historical Perspectives	5
I.5 Chapter Synopses	9
<i>References</i>	11

II Review of Least Squares Data Processing and the Kalman Filter Algorithm

II.1 Introduction	13
II.2 Linear Least Squares	14
II.3 Statistical Interpretation of the Least Squares Solution	15
II.4 Inclusion of <i>a Priori</i> Statistics	16
II.5 Recursions for the Least Squares Information Processor	17
II.6 Kalman Filter Data Processing	19
II.7 Potter's Mechanization of the Kalman Algorithm	22
II.8 Computational Considerations Associated with Covariance Data Processing	24
Appendix II.A Proof that an Overdetermined System with Full Rank Has a Nonsingular Normal Matrix	25
Appendix II.B A Matrix Inversion Lemma	26

Appendix II.C	Data Processing Using the Information Matrix	27
Appendix II.D	Data Processing Using the Kalman Algorithm	28
Appendix II.E	Data Processing Using the Potter Algorithm	30
<i>References</i>		31

III Positive Definite Matrices, the Cholesky Decomposition, and Some Applications

III.1	Positive Definite Matrices	33
III.2	Properties of PD Matrices	34
III.3	Matrix Square Roots and the Cholesky Decomposition Algorithm	37
III.4	Rank One Modification of the Cholesky Factorization	44
III.5	Whitening Observation Errors	47
III.6	Observation Errors That Are Pairwise Correlated	49
III.7	Construction of Random Samples Having a Given Covariance	51
Appendix III.A	Upper Triangular Matrix Factorization Algorithm	51
Appendix III.B	FORTRAN Mechanization of the Lower Triangular Cholesky Factorization	54
Appendix III.C	FORTRAN Mechanization of the UDU^T Update	55
<i>References</i>		55

IV Householder Orthogonal Transformations

IV.1	Review of Orthogonal Transformations	57
IV.2	Application of Orthogonal Matrices to the Least Squares Problem	58
IV.3	The Householder Transformation	59
Appendix IV.A	Annihilating the First Column of a Matrix Using the Householder Transformation	63
Appendix IV.B	Solution of the Triangular System $Rx=y$ and Inversion of a Triangular Matrix	64
<i>References</i>		66

V Sequential Square Root Data Processing

V.1	Introduction	68
V.2	The SRIF Data Processing Algorithm	69
V.3	Data Processing Using the $U-D$ Covariance Factorization	76
V.4	Sequential Data Processing Algorithm Computation Counts and Comparisons	82
V.5	Filter Algorithm Numerical Deterioration; Some Examples	90
Appendix V.A	$U-D$ and Upper Triangular $P^{1/2}$ FORTRAN Mechanizations	100
Appendix V.B	Arithmetic Operation Counts for Various Data Processing Algorithms	103
<i>References</i>		112

VI Inclusion of Mapping Effects and Process Noise

VI.1	Introduction	113
VI.2	Mapping and the Inclusion of Process Noise into the SRIF	115
VI.3	Mapping and the Inclusion of Process Noise into the Kalman Filter	122
VI.4	Mapping and the Inclusion of Process Noise into the U - D Covariance Filter	124
VI.5	Duality Relationships between Information and Covariance Algorithms	129
	Appendix VI.A FORTRAN Mechanization of the MWG-S Algorithm for Time Updating of the U - D Factors	131
	<i>References</i>	133

VII Treatment of Biases and Correlated Process Noise

VII.1	Introduction	135
VII.2	SRIF Data Processing for a System Composed Partly of Biases and Partly of Correlated Process Noise	136
VII.3	SRIF Mapping Mechanization Including the Effects of Correlated Process Noise	141
VII.4	U - D Mapping Mechanization to Account for Bias Parameters and Correlated Process Noise	145
VII.5	Computational Considerations	149
	Appendix VII.A Exponentially Correlated Process Noise	150
	Appendix VII.B A FORTRAN Mechanization of the Epoch State SRIF	152
	<i>References</i>	161

VIII Covariance Analysis of Effects Due to Mismodeled Variables and Incorrect Filter a Priori Statistics

VIII.1	Introduction	162
VIII.2	Consider Analyses for the Batch (Constant Parameter) Filter	165
VIII.3	Considering the Effects of Unestimated Parameters in the Kalman Filter	171
VIII.4	State Variable Augmentation to Evaluate the Effects of Mismodeled Colored Noise	178
	<i>References</i>	182

IX SRIF Error Analysis of Effects Due to Mismodeled Variables and Incorrect Filter a Priori Statistics

IX.1	Introduction	183
IX.2	Evaluating the Effects of Incorrect Filter <i>a Priori</i> Statistics	185
IX.3	Considering the Effects of Unmodeled Biases	191

Appendix IX.A	Processing of the Augmented Information Array Columns	202
Appendix IX.B	The Augmented Array SRIF Error Analysis Algorithm	204
<i>References</i>		210

X Square Root Information Smoothing

X.1	Introduction	211
X.2	Square Root and Non-Square Root Smoother Recursions	214
X.3	The Treatment of Biases in Square Root Smoothing	217
Appendix X.A	Kalman Filter Related Smoothers	220
Appendix X.B	Smoother Recursions Corresponding to the Epoch State SRIF	225
<i>References</i>		231

<i>Bibliography</i>	233
<i>Index</i>	237