Contents

Prefa	ceVII
Gene	ral notationxiv
Chap	oter I. Preliminaries1
1.	Basic notions and notations, 1
2.	Probability measures on a metric space, 2
3.	Expectations, conditional expectations and regular conditional probabilities, 11
4.	Continuous stochastic processes, 16
5.	Stochastic processes adapted to an increasing family of sub σ-fields, 20
6.	Martingales, 25
7.	Brownian motions, 40
8.	
9.	Point processes and Poisson point processes, 43
Chaj	oter II. Stochastic integrals and Itô's formula45
1.	Itô's definition of stochastic integrals, 45
2.	Stochastic integrals with respect to martingales, 53
3.	Stochastic integrals with respect to point processes, 59
4.	Semi-martingales, 63
5.	Itô's formula, 66
6.	Martingale characterization of Brownian motions and Poisson point
	processes, 73
7.	Representation theorem for semi-martingales, 84
Cha	pter III. Stochastic calculus97
1.	The space of stochastic differentials, 97
2.	Stochastic differential equations with respect to quasimartingales, 103
3.	Moment inequalities for martingales, 110
4.	Some applications of stochastic calculus to Brownian motions, 113
	4.1. Brownian local time
	4.2. Reflecting Brownian motion and the Skorohod equation
	4.3. Excursions of Brownian motion

xii CONTENTS

4.4. Some limit theorems for occupation times of Brownian motion5. Exponential martingales, 140
Chapter IV. Stochastic differential equations145
 Definition of solutions, 145 Existence theorem, 153 Uniqueness theorem, 164 Solution by transformation of drift and by time change, 176 Diffusion processes, 188 Diffusion processes generated by differential operators and stochastic differential equations, 198 Stochastic differential equations with boundary conditions, 203 Examples, 218 Stochastic differential equations with respect to Poisson point processes, 230
Chapter V. Diffusion processes on manifolds233
 Stochastic differential equations on manifolds, 233 Flow of diffeomorphisms, 239 Heat equation on a manifold, 254 Non-degenerate diffusions on a manifold and their horizontal lifts, 260 Stochastic parallel displacement and heat equation for tensor fields, 282 The case with boundary conditions, 289 Malliavin's stochastic calculus of variation for Wiener functionals, 322 The case of stochastic differential equations and hypoellipticity problem of heat equations, 334
Chapter VI. Theorems on comparison and approximation and their applications
 A comparison theorem for one-dimensional Itô processes, 352 An application to an optimal control problem, 356 Some results on one-dimensional diffusion processes, 361 Comparison theorem for one-dimensional projection of diffusion processes, 367
 Applications to diffusions on Riemannian manifolds, 375 Stochastic line integrals along the paths of diffusion processes, 382 Approximation theorems for stochastic integrals and stochastic differential equations, 392 The support of diffusion processes, 429
9. Asymptotic evaluation of the diffusion measure for tubes around a smooth curve, 444 Bibliography