TABLE OF CONTENTS

Preface	X
Introduction	
Chapter 1. Definition, Existence, and Uniqueness	
of the Brownian Motion	
1. The Basic Concept	4
2. Construction by Gaussian Interpolation	Ć
3. Construction as a Limit of Random Walks	9
4. Construction by Trigonometric Series	12
5. Uniqueness Questions and Notation	14
Chapter 2. Initial Features of the Process	
1. Equivalence Transformations	19
2. Law of the Iterated Logarithm	22
3. Quadratic Variation	23
4. Modulus of Continuity	25
5. First Passage Times and Pointwise Recurrence	27
6. Hitting Probabilities $(r = 1)$	29
Chapter 3. General Markovian Methods	
3.1. Analytic Methods	31
1. Brownian Transition Densities and Semigroups	31
2. General Markov Semigroups	33
3. Infinitesimal Generators	35
4. Brownian Generators and Resolvents	36
3.2. Probabilistic Methods	38
1. Markov Properties	39
2. Stopping Times and Strong Markov Properties	42
3. Zero or One Laws	50
4. Hitting Times and Dynkin's Formula	52
5. Foreseeability of Stopping Times	55
6. Additive Functionals	57
Chapter 4. Absorbing, Killing, and Time Changing:	
The Classical Cases	
4.1. Absorption	61
1. Two Absorbing Points	61

viii CONTENTS

2.	Space-Time Process and the Heat Equation	63
3.	The Dirichlet Problem in R'	66
4.	The Heat Equation in R'	68
5.	Moments of Passage Times	70
4.2.	Killing	71
1.	Killing at a Boundary	71
2.	Killing by a Continuous Additive Functional	73
3.	The Method of Kac and Rosenblatt	76
4.	Some Sojourn Time Distributions	82
4.3.	Time Changing	89
1.	Sectionally Continuous Coefficients	90
2.	The Corresponding Diffusions on (a, b)	91
3.	The Ornstein-Uhlenbeck Velocity Process	96
4.	Stochastic Differential Equations (Heuristic)	98
5.	Continuous State Branching Processes	100
6.	The Bessel Processes	102
7.	Transience, Neighborhood Recurrence, and Passage Times	103
Chapte	r 5. Local Times, Excursions, and Absolute	
	Sample Path Properties	
5.1.	Local Time: Extrinsic Construction	107
1.	The Skeletal Random Walk	107
2.	The Limit Diffusion	110
3.	Trotter's Theorem and Local Time as a Family	
-	of Additive Functionals	115
5.2.	Brownian Excursions	120
1.	The Brownian Flow	120
2.	The Normalized Excursion	122
3.	Probabilistic Structure of an Excursion	123
4.	Distribution of the Maximum	125
5.3.	The Zero Set and Intrinsic Local Time	127
1.	Distribution of the Zeros	127
2.	Construction of Process from Zeros and Excursions	128
3.	P. Lévy's Equivalence $(Y_1(t), M(t)) \equiv (B(t) , 2s(t, 0))$	130
4.	Passage Time Process as Subordinator	132
5.	The "Mesure du Voisinage" and Local Time	135
6.	The General Sojourn Density Diffusions	137
7.	Local Times of Diffusions	139
5.4.	Some Absolute Sample Path Properties	142
1.	Upper and Lower Classes Locally	142
2.	Lower Escape Rates	147
3.	Global Upper and Lower Moduli	148
4.	Measure of the Range $(r \ge 2)$	149

CONTENTS	i
5. Total Path Variation $(r = 1)$	14
6. Absence of Differentiability or Times of Increase	150
Chapter 6. Boundary Conditions for Brownian Motion $(r = 1)$	
1. Brownian Motions on [0, ∞): Generators	153
2. Construction of the Processes	15
3. Brownian Motions on [0, 1]	162
4. Green Functions and Eigenfunction Expansions	163
Chapter 7. Nonsingular Diffusion in R ¹	
7.1. The Deductive Approach	169
1. First Passage Times and Semigroups	170
2. Local Infinitesimal Generators	173
7.2. The Constructive Approach	170
7.3. Conservative Boundary Conditions	183
7.4. Nonconservative Diffusion	186
1. The General Continuous Additive Functional	187
2. The General Killed Diffusion	10

195 199

Bibliography Index