

Contents

1	Simple Random Walk	11
1.1	Introduction	11
1.2	Local Central Limit Theorem	12
1.3	Strong Markov Property	19
1.4	Harmonic Functions, Dirichlet Problem	21
1.5	Green's Function, Transient Case	28
1.6	Recurrent Case	37
1.7	Difference Estimates and Harnack Inequality	41
2	Harmonic Measure	47
2.1	Definition	47
2.2	Capacity, Transient Case	51
2.3	Capacity, Two Dimensions	57
2.4	Example: Line Segment	62
2.5	Upper Bounds for Harmonic Measure	76
2.6	Diffusion Limited Aggregation	82
3	Intersection Probabilities	87
3.1	Introduction	87
3.2	Preliminaries	88
3.3	Long Range Intersections	93
3.4	Upper Bound in Four Dimensions	97
3.5	Two-Sided Walks	102
3.6	Upper Bound for Two-Sided Walks	105
3.7	One-sided Walks	113
4	Four Dimensions	115
4.1	Introduction	115
4.2	Two-sided Walks	116
4.3	Long-range Intersections	121

4.4	One-sided Walks	128
4.5	Three Walks in Three Dimensions	134
5	Two and Three Dimensions	139
5.1	Intersection Exponent	139
5.2	Intersections of Brownian Motions	141
5.3	Equivalence of Exponents	146
5.4	Variational Formulation	149
5.5	Lower Bound in Two Dimensions	152
5.6	Upper Bound	155
6	Self-Avoiding Walks	163
6.1	Introduction	163
6.2	Connective Constant	164
6.3	Critical Exponents	165
6.4	Edwards Model	170
6.5	Kinetically Growing Walks	175
6.6	Monte Carlo Simulations	178
7	Loop-Erased Walk	183
7.1	Introduction	183
7.2	Erasing Loops	184
7.3	Loop-erased Walk	186
7.4	Two Dimensions	188
7.5	Estimates on Amount Erased	191
7.6	Growth Rate in Low Dimensions	199
7.7	High Dimensions	201