

# Contents

## PREFACE IX

### Part A: Simple Markovian Models

#### CHAPTER I: MARKOV CHAINS 3

1. *Preliminaries* 3
2. *Aspects of renewal theory in discrete time* 6
3. *Stationarity* 10
4. *Ergodic theory* 14
5. *Harmonic functions, martingales, and recurrence/transience criteria* 17
6. *Foundations of the general theory of Markov processes* 20

#### CHAPTER II: MARKOV JUMP PROCESSES 27

1. *Basic structure* 27
2. *The minimal construction* 29
3. *The intensity matrix* 32
4. *Ergodic theory* 37
5. *Time reversibility* 42

#### CHAPTER III: QUEUEING THEORY AT THE MARKOVIAN LEVEL 45

1. *Generalities* 45
2. *General birth–death processes* 55
3. *Birth–death processes as queueing models* 58
4. *Poisson departure processes and series of queues* 63
5. *Queueing networks* 65
6. *The phase method* 71
7. *Lindley processes in discrete time* 78
8. *Random walks and Lindley processes in continuous time* 82

- 9. *Time-dependent properties of  $M/M/1$*  86
- 10. *Waiting times and queue disciplines in  $M/M/1$*  96

### **Part B: Basic Mathematical Tools**

#### **CHAPTER IV: BASIC RENEWAL THEORY 105**

- 1. *Renewal processes* 105
- 2. *Renewal equations and the renewal measure* 109
- 3. *Stationary renewal processes* 115
- 4. *The renewal theorem in its equivalent versions* 118
- 5. *Proof of the renewal theorem* 122

#### **CHAPTER V: REGENERATIVE PROCESSES 125**

- 1. *Basic limit theory* 125
- 2. *First examples and applications* 129
- 3. *Time-average properties* 135

#### **CHAPTER VI: FURTHER TOPICS IN RENEWAL THEORY AND REGENERATIVE PROCESSES 140**

- 1. *Spread-out distributions* 140
- 2. *The coupling method* 143
- 3. *Markov processes: regeneration and Harris recurrence* 150
- 4. *Second-order properties* 158
- 5. *Excessive and defective renewal equations* 161

#### **CHAPTER VII: RANDOM WALKS 165**

- 1. *Basic definitions* 165
- 2. *Ladder processes and classification* 167
- 3. *Wiener-Hopf factorization* 172
- 4. *Transform identities* 174

### **Part C: Special Models and Methods**

#### **CHAPTER VIII: STEADY-STATE PROPERTIES OF $GI/G/1$ 181**

- 1. *Notation. The actual waiting time* 181
- 2. *The moments of the actual waiting time* 184
- 3. *The virtual waiting time* 187

4. *Queue length processes* 191
5. *The robustness of the actual waiting time* 194
6. *Heavy traffic limit theorems* 196

#### CHAPTER IX: EXPLICIT EXAMPLES IN THE THEORY OF RANDOM WALKS AND SINGLE-SERVER QUEUES 201

1. *Ascending ladder heights. GI/M/1* 201
2. *Descending ladder heights. M/G/1* 205
3. *Imbedded Markov chain analysis for GI/M/1* 207
4. *Imbedded Markov chain analysis for M/G/1* 210
5. *More on lattice distributions* 213
6. *Phase-type distributions* 216

#### CHAPTER X: MULTIDIMENSIONAL METHODS 223

1. *Non-negative matrices* 223
2. *Markov renewal theory* 227
3. *Semi-regenerative processes* 232
4. *Random walks on a Markov chain* 235
5. *Matrix-geometric stationary distributions* 239

#### CHAPTER XI: MANY-SERVER QUEUES 246

1. *Comparisons with GI/G/1* 246
2. *Regeneration and existence of limits* 249
3. *The GI/M/s queue* 253

#### CHAPTER XII: CONJUGATE PROCESSES 257

1. *Conjugate random walks* 257
2. *The saddle-point method. Relaxation-time approximations* 259
3. *Continuous time. The inverse Gaussian distribution* 262
4. *The fundamental identity of sequential analysis* 266
5. *The Cramér-Lundberg approximation* 269
6. *Siegmund's corrected heavy traffic approximations* 272
7. *Simulating a conjugate process* 276

#### CHAPTER XIII: INSURANCE RISK, DAM AND STORAGE MODELS 280

1. *Insurance risk models* 280
2. *Ruin probability approximations* 283
3. *Compound Poisson dams with general release rule* 287

4. *Some examples* 295
5. *Further dam and storage models* 297

APPENDIX: SELECTED BACKGROUND AND NOTATION 301

- A1. *Some notation* 301
- A2. *Polish spaces and weak convergence* 302
- A3. *Sample path properties* 303
- A4. *Point processes* 305
- A5. *Stochastic ordering* 305
- A6. *Total variation convergence* 306
- A7. *Transforms* 306
- A8. *Stopping times and Wald's identity* 306
- A9. *Discrete skeletons* 307

REFERENCES 308

INDEX 316