

Table of Contents

	Page
I. INTRODUCTION	1
1.1. General Remarks	1
1.2. The Robbins-Monro Process	4
1.3. A "Continuous" Process Version of Section 2	7
1.4. Regulation of a Dynamical System; a simple example	8
1.5. Function Minimization: The Kiefer-Wolfowitz Procedure	9
1.6. Constrained Problems	11
1.7. An Economics Example	15
II. CONVERGENCE w.p.1 FOR UNCONSTRAINED SYSTEMS	19
2.1. Preliminaries and Motivation	19
2.2. The Robbins-Monro and Kiefer-Wolfowitz Algorithms: Conditions and Discussion	28
2.3. Convergence Proofs for RM and KW-like Procedures	38
2.3.1. A Basic RM-like Procedure	38
2.3.2. One Dimensional RM and Accelerated RM Procedures	43
2.3.3. A Continuous Parameter RM Procedure	47
2.3.4. The Basic Kiefer-Wolfowitz Procedure	51
2.3.5. Random Directions KW Methods	58
2.4. A General Robbins-Monro Process: "Exogenous Noise"	63
2.4.1. The Case of Bounded $h(\cdot, \cdot)$	64
2.4.2. Unbounded $h(\cdot, \cdot)$: Exogenous Noise	70
2.5. A General RM Process; State Dependent Noise	71
2.5.1. Extensions and Localizations of Theorem 2.5.2	77
2.6. Some Applications	87
2.7. Mensov-Rademacher Estimates	98

	Page
III. WEAK CONVERGENCE OF PROBABILITY MEASURES	100
IV. WEAK CONVERGENCE FOR UNCONSTRAINED SYSTEMS	106
4.1. Conditions and General Discussion	108
4.2. The Robbins-Monro and Kiefer-Wolfowitz Procedures	117
4.2.1. The Basic Robbins-Monro Procedure	117
4.2.2. The One-Dimensional Robbins-Monro Procedure	123
4.2.3. The Kiefer-Wolfowitz Procedure	124
4.2.4. A Case Where the Limit Satisfies a Generalized ODE	126
4.2.5. A Continuous Parameter KW Procedure	129
4.3. A General Robbins-Monro Process: Exogenous Noise	132
4.4. A General RM Process: State Dependent Noise	139
4.5. The Identification Problem	143
4.6. A Counter-Example to Tightness	146
4.7. Boundedness of $\{X_n\}$ and Tightness of $\{X^n(\cdot)\}$	150
V. CONVERGENCE w.p.1 FOR CONSTRAINED SYSTEMS	158
5.1. A Penalty-Multiplier Algorithm for Equality Constraints	159
5.1.1. A Basic RM-like Algorithm, Conditions and Discussion	159
5.1.2. The Noise Condition, Discussion and Generalization	163
5.1.3. Boundedness of $\{X_n\}$	171
5.1.4. Proof of the Main Theorem	173
5.1.5. Constrained Function Minimization and Other Extensions	174
5.2. A Lagrangian Method for Inequality Constraints	177
5.2.1. The Algorithm and Conditions	177
5.2.2. The Convergence Theorem	182

	Page
5.2.3. A Non-Convergent but Useful Algorithm	186
5.2.4. An Application to the Identification Problem	188
5.3. A Projection Algorithm	189
5.4. A Penalty-Multiplier Method for Inequality Constraints	196
VI. WEAK CONVERGENCE: CONSTRAINED SYSTEMS	209
6.1. A Multiplier Type Algorithm for Equality Constraints	209
6.1.1. Boundedness of $\{X_n\}$	210
6.1.2. The Noise Condition, Discussion	212
6.1.3. The Convergence Theorem	216
6.2. The Lagrangian Method	218
6.3. A Projection Algorithm	223
6.4. A Penalty-Multiplier Algorithm for Inequality Constraints	225
VII. RATES OF CONVERGENCE	232
7.1. The Problem Formulation	233
7.2. Conditions and Discussions	239
7.3. Rates of Convergence for Case 1, the KW Algorithm	244
7.4. Discussion of Rates of Convergence for Two KW Algorithms	252
BIBLIOGRAPHY	257
INDEX	260