

CONTENTS

PART I. STOCHASTIC PROCESSES

I.1. INTRODUCTION	3
I.2. MARKOV CHAINS WITH A DISCRETE TIME PARAMETER.	6
I.2.1. Definition of the process	6
I.2.2. Classification of states	12
I.2.3. Ergodic properties of aperiodic irreducible Markov chains.	18
I.2.4. Foster's criteria	21
I.2.5. Taboo probabilities.	31
I.3. MARKOV CHAINS WITH A CONTINUOUS TIME PARAMETER	34
I.3.1. Definition of the process	34
I.3.2. The Q-matrix	37
I.3.3. Types of states and ergodic properties.	50
I.3.4. Foster's criteria	56
I.4. BIRTH AND DEATH PROCESSES.	61
I.4.1. Introduction.	61
I.4.2. The Poisson process	68
I.4.3. Death process with constant death rate	76
I.4.4. Birth and death process with constant birth and death rates	77
I.5. DERIVED MARKOV CHAINS	86
I.5.1. Introduction.	86
I.5.2. Derived Poisson process.	89
I.5.3. Derived death process	90
I.6. RENEWAL THEORY AND REGENERATIVE PROCESSES	95
I.6.1. Introduction.	95
I.6.2. Renewal theorems	100

I.6.3.	Past lifetime and residual lifetime	108
I.6.4.	Regenerative processes	115
I.6.5.	Some special renewal distributions	135
I.6.6.	Fluctuation theory	141

PART II. THE SINGLE SERVER QUEUE

II.1.	THE MODEL	159
II.1.1.	Introduction	159
II.1.2.	Description of the model	162
II.1.3.	Some general theorems and relations	167
II.2.	THE QUEUEING SYSTEM $M/M/1$	175
II.2.1.	The number of customers in the system	175
II.2.2.	The busy period	187
II.2.3.	The waiting time	193
II.2.4.	The departure process	195
II.3.	THE QUEUEING SYSTEM $G/M/1$	203
II.3.1.	Introduction	203
II.3.2.	Description of the process for $t \rightarrow \infty$	204
II.3.3.	The entrance and return time distributions	211
II.3.4.	The transition probabilities	220
II.3.5.	The busy period	225
II.3.6.	The waiting time	229
II.3.7.	The departure process	231
II.4.	THE QUEUEING SYSTEM $M/G/1$	234
II.4.1.	Introduction	234
II.4.2.	The imbedded Markov chain z_n for $n \rightarrow \infty$	236
II.4.3.	Transition probabilities and first passage times	239
II.4.4.	The busy period	249
II.4.5.	The waiting time	253
II.4.6.	The departure process	264
II.5.	THE QUEUEING SYSTEM $G/G/1$	267
II.5.1.	Introduction	267
II.5.2.	Pollaczek's integral equation	271
II.5.3.	The joint distribution of w_n, d_n, a_{n-1} and b_{n-1}	275
II.5.4.	The actual waiting time	277
II.5.5.	The busy period, the idle period and the busy cycle	283
II.5.6.	The virtual waiting time and the idle time	290
II.5.7.	The queue length	298
II.5.8.	Integral representations	303
II.5.9.	Truncated service time distribution, the system $G/G_c/1$	307

II.5.10.	The queueing system $G/K_n/1$	321
II.5.11.	The queueing system $K_m/G/1$	329
II.6.	SOME SPECIAL METHODS	332
II.6.1.	Introduction	332
II.6.2.	The method of the supplementary variable for the $M/G/1$ queue.	332
II.6.3.	Lindley's integral equation	337
II.6.4.	The method of collective marks.	340
II.6.5.	The phase method and its variants	341
II.6.6.	The combinatorial method	349
 PART III. SOME VARIANTS OF THE SINGLE SERVER QUEUE 		
III.1.	INTRODUCTION	367
III.2.	THE BULK QUEUE $G/G/1$	369
III.2.1.	The model	369
III.2.2.	Group arrivals, individual service for $G/G/1$	371
III.2.3.	Group arrivals, individual service for $M/G/1$	373
III.2.4.	Group arrivals, batch service for $M/G/1$	376
III.2.5.	The transportation problem	392
III.2.6.	The bulk $M/G/1$ with accessible batches.	400
III.2.7.	The queue $G/M/1$ with batch service	401
III.2.8.	The bulk queue $G/M/1$ in inventory control	408
III.2.9.	The $G/G/1$ bulk queue with single arrivals and delayed service	412
III.3.	PRIORITY DISCIPLINES FOR A SINGLE SERVER QUEUE	415
III.3.1.	Introduction.	415
III.3.2.	Waiting time for "last come, first served"	418
III.3.3.	Random service for $M/G/1$	427
III.3.4.	Random service for $G/M/1$	431
III.3.5.	Completion times	434
III.3.6.	Pre-emptive resume priority for $M/G/1$ with two priority levels.	437
III.3.7.	Pre-emptive repeat priority for $M/G/1$ with two priority levels	441
III.3.8.	Non break-in priority for $M/G/1$ with two and three priority levels	442
III.3.9.	General pre-emptive resume priority for $M/G/1$	450
III.3.10.	General non break-in priority for $M/G/1$	452
III.3.11.	Non break-in random priorities for $M/G/1$	458
III.3.12.	Comparison of variances for various service disciplines for $M/G/1$	463
III.4.	UNIFORMLY BOUNDED ACTUAL WAITING TIME.	466
III.4.1.	Introduction.	466
III.4.2.	The time dependent solution.	467
III.4.3.	The stationary distribution	475
III.4.4.	Entrance times and return times	479

III.4.5.	Application to the system $M/G/1$.	483
III.4.6.	Application to the system $G/M/1$.	489
III.4.7.	Application to the system $M/M/1$.	493
III.5.	THE FINITE DAM; UNIFORMLY BOUNDED VIRTUAL WAITING TIME	495
III.5.1.	Introduction.	495
III.5.2.	The time dependent solution.	497
III.5.3.	The stationary distributions	506
III.5.4.	Entrance times and return times	512
III.5.5.	The finite dam model $M/G/1$	517
III.5.6.	The finite dam model $G/M/1$	529
III.5.7.	Application to the system $M/M/1$	533
III.5.8.	The distribution of crossings of a level K for the infinite dam $M/G/1$	534
III.5.9.	The integral equation for the finite dam $M/G/1$	540
III.5.10.	The dam $M/G/1$ with non-constant release.	543
III.5.11.	The dam $G/G/1$ with non-instantaneous input	547
III.6.	THE $M/G/1$ QUEUEING SYSTEM WITH FINITE WAITING ROOM	551
III.6.1.	Introduction.	551
III.6.2.	The integral equation and its solution.	552
III.6.3.	Stationary distributions	558
III.6.4.	Entrance times and return times	566
III.6.5.	Return times of a state K with in a busy cycle for the $M/G/1$ queue with infinite waiting room.	575
III.7.	LIMIT THEOREMS FOR SINGLE SERVER QUEUES	583
III.7.1.	Introduction.	583
III.7.2.	Heavy traffic theory	584
III.7.3.	The relaxation time.	588
III.7.4.	Limit theorems for the $M/G/1$ queueing system.	602
III.7.5.	Limit theorems for the $G/M/1$ queueing system.	615
III.7.6.	Behaviour of v_t for $t \rightarrow \infty$ if $a > 1$	619
APPENDIX		622
NOTES ON LITERATURE		629
REFERENCES		639
AUTHOR INDEX		648
INDEX OF NOTATIONS		650
SUBJECT INDEX		653