

# Contents

*From the Preface to the Russian Edition*

vii

I. Fundamental Concepts of the Calculus of Variations . . . . .	1
1. Functionals . . . . .	1
2. Admissible Lines. Function Classes . . . . .	3
3. Nearness of Functions . . . . .	5
4. Classification of Extremums . . . . .	6
5. Euler Equation . . . . .	7
6. Discussion of the Euler Equation . . . . .	13
7. The Legendre Condition . . . . .	16
II. Generalizations of the Simplest Problem of Calculus of Variations . . . . .	20
8. Problems with Variable Endpoints. General Formula for the Variations	20
9. Transversality Conditions . . . . .	22
10. Extremals with Breaks. Weierstrass-Erdmann Conditions . . . . .	24
11. Functionals Dependent on Several Unknown Functions . . . . .	27
12. Functionals Dependent on Higher-Order Derivatives . . . . .	29
13. Conditional Extremum. . . . .	34
14. Isoperimetric Problem . . . . .	37
15. General Lagrange Problem. Maier and Bolza Problems. . . . .	41
16. Variational Problems in Parametric Form . . . . .	43
17. Canonical Form of the Euler Equations . . . . .	45
18. Extremum of a Functional Dependent on a Function of Several Variables . . . . .	46
III. Applying the Euler Equation to the Solution of Engineering Problems . . . . .	50
19. Direct Current Electric Motor . . . . .	50

20. Estimate of the Change in a Functional When the Actual Function Deviates from the Extremal . . . . .	57
21. Reciprocity Principle; Its Boundedness . . . . .	61
22. Selection of the Optimum Gear Ratio. Extremals with a Parameter .	65
23. Electric Load Driver with Time-Dependent Resistance Moment. Boundary Conditions at Infinity . . . . .	72
24. More General Problems of Optimum Control. Electric Drive with a Resistance Moment Dependent on the Velocity, and a Magnetic Flux Dependent on the Armature Current . . . . .	75
<b>IV. Field Theory. Sufficient Conditions for an Extremum . . . . .</b>	<b>83</b>
25. Field of Extremals . . . . .	83
26. Jacobi and Legendre Conditions . . . . .	85
27. Strong Extremum. Weierstrass Condition . . . . .	90
28. Summary of Necessary and Sufficient Conditions for an Extremum .	96
29. Degenerate Functionals . . . . .	99
30. The Work of V. F. Krotov. . . . .	103
<b>V. Extremum Problem with Constraints . . . . .</b>	<b>115</b>
31. Problems with Constraints in Classical Calculus of Variations . . . . .	115
32. Linear Optimum Control Problems . . . . .	126
33. The Maximum Principle . . . . .	133
34. Synthesis of an Optimum Control . . . . .	138
35. Dynamic Programming . . . . .	141
36. Nonstandard Functionals . . . . .	144
37. Appropriate Methods of Solution . . . . .	147
<b>VI. Examples of the Application of Variational Methods . . . . .</b>	<b>151</b>
38. Optimum Control of DC Electric Motors with Velocity and Armature Current Constraints . . . . .	151
39. Control Assuring Minimum Rated Generator Power (Example with a Nonstandard Functional) . . . . .	157
40. Control of a Compound with Independent Excitation in the Armature and Excitation Loops . . . . .	159
41. Control with a Voltage Constraint . . . . .	162
42. Determination of the Maximum Allowable Dynamic Effect . . . . .	166
43. Control of the Excitation of a Synchronous Machine Assuring the Highest Degree of Stability . . . . .	170

44. Optimum Control of Locomotive Motion . . . . .	175
45. Amplitude and Frequency Control of Asynchronous Electric Motors . . . . .	182
Appendix I: Historical Survey . . . . .	194
Appendix II: Glossary . . . . .	201
References . . . . .	204
<i>Author Index</i> . . . . .	213
<i>Subject Index</i> . . . . .	215