

Contents

	Page
Chapter I. The Galton-Watson branching process	1
1. Historical remarks	1
2. Definition of the Galton-Watson process	3
2.1. Mathematical description of the Galton-Watson process	4
2.2. Generating functions	5
3. Basic assumptions	5
4. The generating function of Z_n	5
5. Moments of Z_n	6
6. The probability of extinction	7
6.1. Instability of Z_n	8
7. Examples	9
7.1. Fractional linear generating functions	9
7.2. Another example	10
7.3. Survival of animal families or genes	10
7.4. Electron multipliers	11
8. Asymptotic results when $m > 1$	11
8.1. Convergence of the sequence $\{Z_n/m^n\}$	12
8.2. The distribution of W	15
8.3. Asymptotic form of $P(Z_n = 0)$	16
8.4. Local limit theorems when $m > 1$	17
8.5. Examples	17
9. Asymptotic results when $m < 1$	18
10. Asymptotic results when $m = 1$	19
10.1. Form of the iterates f_n when $m = 1$	19
10.2. The probability of extinction when n is large	21
10.3. Distribution of Z_n when n is large	21
10.4. The past history of a surviving family when $m = 1$	22
11. Stationarity of Z_n	22
11.1. Stationary probabilities	23
11.2. Stationary measures	23
11.3. Existence of a stationary measure for the Galton-Watson process .	24
11.4. The question of uniqueness	27
11.5. The form of the π_i when i is large, for the case $m = 1$	27
11.6. Example: fractional linear generating function	28
12. An application of stationary measures	29
13. Further results on the Galton-Watson process and related topics	31
13.1. Joint generating function of the various generations	31
13.2. Distribution of $Z_0 + Z_1 + \dots + Z_n$ and of $Z = Z_0 + Z_1 + \dots$.	32
13.3. The time to extinction	32
13.4. Estimation of parameters	32
13.5. Variable generating function	33
13.6. Trees, etc.	33
13.7. Percolation processes	33

	Page
Chapter II. Processes with a finite number of types	34
1. Introduction	34
2. Definition of the multitype Galton-Watson process	35
3. The basic result for generating functions	36
4. First and second moments; basic assumption	36
5. Positivity properties	37
6. Transience of the nonzero states	38
7. Extinction probability	40
8. A numerical example	43
9. Asymptotic results for large n	44
9.1. Results when $\varrho < 1$	44
9.2. The case $\varrho = 1$	44
9.3. Results when $\varrho > 1$	44
10. Processes that are not positively regular	45
10.1. The total number of objects of various types	47
11. An example from genetics	47
12. Remarks	49
12.1. Martingales	49
12.2. The expectation process	49
12.3. Fractional linear generating functions	49
Chapter III. The general branching process	50
1. Introduction	50
2. Point-distributions and set functions	51
2.1. Set functions	51
3. Probabilities for point-distributions	52
3.1. Rational intervals, basic sets, cylinder sets	55
3.2. Definition of a probability measure on the point-distributions	55
4. Random integrals	56
5. Moment-generating functionals	56
5.1. Properties of the MGF of a random point-distribution	57
5.2. Alternative formulation	59
6. Definition of the general branching process	59
6.1. Definition of the transition function	60
6.2. Notation	61
7. Recurrence relation for the moment-generating functionals	61
8. Examples	62
8.1. The nucleon cascade and related processes	62
8.2. A one-dimensional neutron model	63
9. First moments	64
9.1. Expectations of random integrals	65
9.2. First moment of Z_n	65
10. Existence of eigenfunctions for M	66
10.1. Eigenfunctions and eigenvalues	67
11. Transience of Z_n	68
12. The case $\varrho \leq 1$	70
12.1. Limit theorems when $\varrho \leq 1$	70
13. Second moments	70
13.1. Expectations of random double integrals	71
13.2. Recurrence relation for the second moments	71
13.3. Asymptotic form of the second moment when $\varrho > 1$	72
13.4. Second-order product densities	72

	Page
14. Convergence of Z_n/ϱ^n when $\varrho > 1$	72
15. Determination of the extinction probability when $\varrho > 1$	73
16. Another kind of limit theorem	74
17. Processes with a continuous time parameter	75
Appendix 1	76
Appendix 2	77
Appendix 3	78
Chapter IV. Neutron branching processes (one-group theory, isotropic case)	80
1. Introduction	80
2. Physical description	81
3. Mathematical formulation of the process	81
3.1. Transformation probabilities	82
3.2. The collision density	82
3.3. Definition of the branching process	83
4. The first moment	84
5. Criticality	84
6. Fluctuations; probability of extinction; total number in the critical case	85
6.1. Numerical example	86
6.2. Further discussion of the example	87
6.3. Total number of neutrons in a family when the body is critical	88
7. Continuous time parameter	88
7.1. Integral equation treatment	89
8. Other methods	91
9. Invariance principles	91
10. One-dimensional neutron multiplication	92
Chapter V. Markov branching processes (continuous time)	93
1. Introduction	93
2. Markov branching processes	95
3. Equations for the probabilities	97
3.1. Existence of solutions	97
3.2. The question of uniqueness	98
3.3. A lemma	98
4. Generating functions	99
4.1. Condition that the probabilities add to 1	100
5. Iterative property of F_t ; the imbedded Galton-Watson process	100
5.1. Imbedded Galton-Watson processes	101
5.2. Fractional iteration	101
6. Moments	103
7. Example: the birth-and-death process	103
8. YULE's problem	105
9. The temporally homogeneous case	106
10. Extinction probability	107
11. Asymptotic results	108
11.1. Asymptotic results when $h'(1) < 1$	108
11.2. Asymptotic results when $h'(1) = 1$	109
11.3. Asymptotic results when $h'(1) > 1$	109
11.4. Extensions	110
12. Stationary measures	110
13. Examples	112
13.1. The birth-and-death process	112

	Page
13.2. Another example	112
13.3. A case in which $F_1(1, t) < 1$	112
14. Individual probabilities	112
15. Processes with several types	113
15.1. Example: the multiphase birth process	114
15.2. Chemical chain reactions	115
16. Additional topics	116
16.1. Birth-and-death processes (generalized)	116
16.2. Diffusion model	116
16.3. Estimation of parameters	117
16.4. Immigration	117
16.5. Continuous state space	118
16.6. The maximum of $Z(t)$	118
Appendix 1	119
Appendix 2	120
Chapter VI. Age-dependent branching processes	121
1. Introduction	121
2. Family histories	122
2.1. Identification of objects in a family	123
2.2. Description of a family	123
2.3. The generations	124
3. The number of objects at a given time	125
4. The probability measure P	126
5. Sizes of the generations	127
5.1. Equivalence of $\{\zeta_n > 0, \text{ all } n\}$ and $\{Z(t) > 0, \text{ all } t\}$; probability of extinction	128
6. Expression of $Z(t, \omega)$ as a sum of objects in subfamilies	129
7. Integral equation for the generating function	130
7.1. A special case	131
8. The point of regeneration	131
9. Construction and properties of $F(s, t)$	132
9.1. Another sequence converging to a solution of (7.3)	133
9.2. Behavior of $F(0, t)$	133
9.3. Uniqueness	134
9.4. Another property of F	135
9.5. Calculation of the probabilities	135
10. Joint distribution of $Z(t_1), Z(t_2), \dots, Z(t_k)$	136
11. Markovian character of Z in the exponential case	136
12. A property of the random functions; nonincreasing character of $F(1, t)$	138
13. Conditions for the sequel; finiteness of $Z(t)$ and $\mathcal{E}Z(t)$	138
14. Properties of the sample functions	139
15. Integral equation for $M(t) = \mathcal{E}Z(t)$; monotone character of M	140
15.1. Monotone character of M	141
16. Calculation of M	141
17. Asymptotic behavior of M ; the Malthusian parameter	142
18. Second moments	144
19. Mean convergence of $Z(t)/n_1 e^{\alpha t}$	145
20. Functional equation for the moment-generating function of W	146
21. Probability 1 convergence of $Z(t)/n_1 e^{\alpha t}$	147
22. The distribution of W	149

	Page
23. Application to colonies of bacteria	150
24. The age distribution	151
24.1. The mean age distribution	152
24.2. Stationarity of the limiting age distribution.	153
24.3. The reproductive value.	153
25. Convergence of the actual age distribution.	154
26. Applications of the age distribution.	156
26.1. The mitotic index	156
26.2. The distribution of life fractions	157
27. Age-dependent branching processes in the extended sense	157
28. Generalizations of the mathematical model	158
28.1. Transformation probabilities dependent on age	158
28.2. Correlation between sister cells	158
28.3. Multiple types	158
29. Age-dependent birth-and-death processes	159
Appendix	161

Chapter VII. Branching processes in the theory of cosmic rays (electron-photon cascades) 164

1. Introduction.	164
2. Assumptions concerning the electron-photon cascade	166
2.1. Approximation <i>A</i>	167
2.2. Approximation <i>B</i>	167
3. Mathematical assumptions about the functions <i>q</i> and <i>k</i>	168
3.1. Numerical values for <i>k</i> , <i>q</i> , and λ ; units	168
3.2. Discussion of the cross sections	169
4. The energy of a single electron (Approximation <i>A</i>)	170
5. Explicit representation of $\varepsilon(t)$ in terms of jumps	171
5.1. Another expression for $\varepsilon(t)$	174
6. Distribution of $X(t) = -\log \varepsilon(t)$ when <i>t</i> is small	175
7. Definition of the electron-photon cascade and of the random variable $N(E, t)$ (Approximation <i>A</i>)	177
7.1. Indexing of the particles	178
7.2. Histories of lives and energies	178
7.3. Probabilities in the cascade; definition of Ω	179
7.4. Definition of $N(E, t)$	180
8. Conservation of energy (Approximation <i>A</i>).	180
9. Functional equations	182
9.1. Introduction	182
9.2. An integral equation	184
9.3. Derivation of the basic equations (11.14) in case $\mu = 0$	184
10. Some properties of the generating functions and first moments	185
11. Derivation of functional equations for f_1 and f_2	187
11.1. Singling out of photons born before Δ	188
11.2. Simplification of equation (11.1).	189
11.3. Limiting form of $f_2(s, E, t + \Delta)$ as $\Delta \downarrow 0$	190
12. Moments of $N(E, t)$	192
12.1. First moments	192
12.2. Second and higher moments	193
12.3. Probabilities	193
12.4. Uniqueness of the solution of (11.14)	193

	Page
13. The expectation process	194
13.1. The probabilities for the expectation process	195
13.2. Description of the expectation process	196
14. Distribution of $Z(t)$ when t is large	198
14.1. Numerical calculation	199
15. Total energy in the electrons	200
15.1. Martingale property of the energy	201
16. Limiting distributions	202
16.1. Case in which $t \rightarrow \infty$, E fixed	202
16.2. Limit theorems when $t \rightarrow \infty$ and $E \rightarrow 0$	203
17. The energy of an electron when $\beta > 0$ (Approximation B)	203
18. The electron-photon cascade (Approximation B)	206
Appendix 1	207
Appendix 2	208
Bibliography	211
Index	225