CONTENTS



Preface			vii		
Prologue			xi		
Volume I					
Chapter 1.	Threshold of Complexity				
	1.	Introduction	1		
	2.	Cellular Automata is a Special Case of CNN	3		
	3.	Every Local Rule is a Cube with Eight Colored Vertices	4		
	4.	Every Local Rule is a Code for Attractors of a Dynamical System	6		
		4.1. Dynamical system for rule 110	78		
		4.2. There are eight attractors for each local rule	79		
	5.	Every Local Rule has a Unique Complexity Index	83		
		5.1. Geometrical interpretation of projection σ and			
		discriminant $w(\sigma)$	83		
		5.2. Geometrical interpretation of transition points of			
		discriminant function $w(\sigma)$	84		
		5.3. Geometrical structure of local rules	86		
		5.4. A local rule with three separation planes	87		
		5.5. Linearly separable rules	95		
		5.6. Complexity index	96		

		5.7. Every local rule is a member of an equivalence class	102
		5.8. Making non-separable from separable rules	110
		5.9. Index 2 is the threshold of complexity	111
Chapter 2.	Universal Neuron		
	1.	Firing and Quenching Patterns	113
	2.	A Universal Neuron	119
	3.	Gallery of One-Dimensional Cellular Automata	121
	4.	Genealogic Classification of Local Rules	190
		4.1. Primary and secondary firing patterns	190
		4.2. Partitioning 256 local rules into 16 gene families	192
		4.3. Each gene family has 16 gene siblings	192
	5.	The Double-Helix Torus	198
		5.1. Algorithm for generating all 16 local rules	
		belonging to each gene family	198
		5.2. "8/24" Distribution pattern in gene siblings	198
		5.3. Coding local rules on a double helix	206
	6.	Explaining and Predicting Pattern Features	210
		6.1. Gallery of gene family patterns	210
		6.2. Predicting the background	210
Chapter 3.	Pr	edicting the Unpredictable	229
	1.	Introduction	229
	2.	Local versus Global Equivalence	233
	3.	Predicting the Unpredictable	241
		3.1. Paritioning 256 local rules into 89 global	
		equivalence classes	241
		3.2. The Vierergruppe \mathcal{V} : Key for defining global	
		transformations \mathbf{T}^{\dagger} , $\overline{\mathbf{T}}$, and \mathbf{T}^{*}	248
		3.3. Proof of global equivalence	259
	4.	Predicting the Predictable	264
		4.1. The rotation group \mathcal{R}	265
		4.2. Local equivalence classes	270
		4.3. Finding all rotations which map any $N \in S^{\mathbf{n}}_{\mathbf{m}}$ to	
		any $N' \in S^{\mathbf{n}}_{\mathbf{m}}$	295
		4.4. Truth-table mapping matrices for the rotation group \mathcal{R}	296
		4.5. Combining transformations from the Vierer gruppe ${\mathcal V}$ and	
		the rotation group ${\cal R}$	338

		Contents	xiii
	4.6. Laterally symmetric interaction group for local		
	equivalence classes	338	
	4.7. Mapping parameter vectors between rule 110 and		
	its locally-equivalent rules	338	
	5. Concluding Remarks	359	
References		361	
Index (for V	'olume I)	363	
Volume II			
Chapter 4.	From Bernoulli Shift to $1/f$ Spectrum	369	
	1. Introduction	369	
	1.1. Computing all 256 rules from one CA difference equation	371	
	2. Mapping Local Rules onto Global Characteristic Functions	371	
	2.1. CA characteristic functions	372	
	2.2. Algorithm for plotting the graph of CA characteristic		
	functions	372	
	2.3. A glimpse of some time- τ characteristic functions $\chi_{\overline{N}}^{\tau}$	373	
	3. Transient Regimes and Attractors	379	
	3.1. Mapping CA attractors onto time- τ maps	382	
	3.2. A gallery of time-1 maps and power spectrum	387	
	3.3. Three general properties of time-1 maps	452	
	3.4. Invertible time- τ maps	453	
	4. Period- k Time-1 Maps: $k = 1, 2, 3$	454	
	4.1. Period-1 rules	454	
	4.2. Period-2 rules	459	
	4.3. Period-3 rules	460	
	4.4. Invariant orbits	463	
	5. Bernoulli σ_{τ} -Shift Rules	463	
	5.1. Gallery of Bernoulli σ_{τ} -shift rules	463	
	5.2. Predicting the dynamic evolution from $\{\beta, \tau\}$	475	
	5.3. Two limiting cases: Period-1 and palindrome rules	487	
	5.4. Resolving the multivalued paradox	491	

	6. P	redictions from Power Spectrum	494
	6.	1. Characteristic features of Bernoulli rules	494
	6.	2. Turing-universal rules: {[110], [124], [137], [193]} exhibit	
		1/f power-frequency characteristics	499
	7. C	oncluding Remarks	499
Chapter 5.	Fract	tals Everywhere	509
	1. C	Characteristic Functions: Global Representation of	
	\mathbf{L}	ocal Rules	509
	1.	.1. Deriving explicit formula for calculating $\chi^1_{\overline{N}}$	511
		.2. Graphs of characteristic functions $\chi^1_{ N }$	528
		.3. Deriving the Bernoulli map from $\chi_{\overline{170}}^{1}$	528
		.4. Deriving inverse Bernoulli map from $\chi^1_{[240]}$	528
		.5. Deriving affine (mod 1) characteristic functions	593
	2. I	Lameray Diagram on on $\chi^1_{[N]}$ Gives Attractor Time-1 Maps	596
	2	2.1. Lameray diagram of 170	597
	2	2.2. Lameray diagram of 240	597
	2	2.3. Lameray diagram of 2	597
	2	2.4. Lameray diagram of 3	597
	2	2.5. Lameray diagram of 46	597
	2	2.6. Lameray diagram of 110	603
	2	2.7. Lameray diagram of 30	607
	3. (Characteristic Functions are Fractals	610
	4. H	Predicting the Fractal Structures	618
	4	1.1. Two-level fractal stratifications	618
		4.1.1. Stratification prediction procedure	623
		4.1.2. Examples illustrating stratification prediction	
		procedure	623
		4.1.3. $\{\Phi_1, \Phi_2, \Phi_3, \Phi_4\}$ — stratified families	625
	4	4.2. Rules having no fractal stratifications	630
	2	4.3. Origin of the fractal structures	630
	5.	Gardens of Eden	634
	6.	Isle of Eden	655
	7.	Concluding Remarks	656

Chapter 6.	Fr	om Time-Reversible Attractors to the Arrow of Time	657
	1.	Recap on Time- τ Characteristic Functions and Return Maps	658
	2.	Rule 62 Has Four Distinct Topological Dynamics	660
		2.1. Period-1 attractor $\Lambda_1(\overline{62})$ and its basin tree	000
		$\mathcal{B}_{I+1}[\Lambda_1(\overline{\ket{62}})]$	667
		2.2. Period-3 "fisle of Eden" orbits $\Lambda_2(\boxed{62})$	675
		2.3. Period-3 attractors $\Lambda_3(\boxed{62})$ and their basin trees	
		$\mathcal{B}_{I+1}[\Lambda_3(\overline{[62]})]$	675
		2.4. Bernoulli σ_{τ} -shift attractors $\Lambda_4(\boxed{62})$ and their basin trees	
		$\mathcal{B}_{I+1}[\Lambda_4(\overline{[62]})]$	676
	3.	Concept of a Time-Reversible	
		Attractor	700
	4.	Time-Reversible Rules	705
		4.1. Relationship between invertible and time-reversible attractors	706
		4.2. Time reversible does not imply invertible	706
		4.3. Time-reversible implies invertible if it is not period-1	707
		4.4. Table of time-reversible rules	707
	5.	There are 84 Time-Reversible Bernoulli $\sigma_{\tau}\text{-Shift Rules}$	715
		5.1. There are 42 time-reversible Bernoulli σ_{τ} -shift rules	
		(with $ \sigma = 1$, $\beta = 2^{\sigma} > 0$, and $\tau = 1$) having only	
		one Bernoulli attractor	715
		5.2. Four canonical Bernoulli shift maps	732
		5.3. There are eight time-reversible time-2 Bernoulli σ_{τ} -shift	
		rules (with $ \sigma = 1$, $\beta = 2^{\sigma} > 0$, and $\tau = 2$) having only one	
		Bernoulli attractor	732
		5.4. There are 32 time-reversible Bernoulli σ_{τ} -shift rules with	
		two invertible attractors	732
		5.5. Composition of 84 time-reversible Bernoulli rules	732
	6.	What Bit Strings Are Allowed in an Attractor or Invariant	
		Orbit?	765
		6.1. Laws governing period-1 bit strings	765
		6.2. Laws governing period-2 bit strings	792
		6.3. Laws governing period-3 bit strings	792
		6.4. Laws governing Bernoulli σ_{τ} -shift bit strings	792
		6.4.1. Shift left or shift right by one bit	792
		6.4.2. Unfolding Bernoulli orbits in complex plane	816

		6.4.3.	Shift left or shift right by one bit and followed by	
			complementation	816
		6.4.4.	Shift left or shift right by one bit every two	
			iterations	827
		6.4.5.	Time-reversible rules with two Bernoulli	
			attractors	827
		6.4.6.	Time-irreversible rules with two Bernoulli	
			attractors	827
		6.4.7.	Time-irreversible rules with three Bernoulli	
			attractors	827
		6.4.8.	Deriving bit string laws for globally-equivalent rules	
			is trivial	827
7.	Mat	hemati	cal Foundation of Bernoulli $\sigma_{ au}$ -Shift Maps	867
	7.1.	Exact	formula for time-1 Bernoulli maps for rules $\boxed{170}$,	
		240,	[85], or [15]	867
		7.1.1.	Exact formula for Bernoulli right-copycat	
			shift map 170	867
		7.1.2.	Exact formula for Bernoulli left-copycat	
			shift map 240	871
		7.1.3.	Exact formula for Bernoulli shift map for 85	873
		7.1.4.	Exact formula for Bernoulli shift map for 15	874
		7.1.5.	Exact formula for Bernoulli shift maps with	
			$\beta = 2^{\sigma}$ (left shift), or $\beta = 2^{-\sigma}$ (right shift), $\sigma = 1$	
			and $\tau = 1$	874
		7.1.6.	Exact formula for Bernoulli shift maps for 184	875
	7.2.	Exact	formula for time- τ Bernoulli maps for rules $\boxed{170}$,	
		240,	85, or 15	876
		7.2.1.	Geometrical interpretation of $time$ - τ maps	876
		7.2.2.	Exact formula for time-2 Bernoulli left-shift map for	
			rule 170	881
		7.2.3.	Exact formula for time-2 Bernoulli right-shift map for	
			rule 240	884
		7.2.4.	Exact formulas for generalized Bernoulli maps	888
		7.2.5.	Analytical proof of period-3 "Isle of Eden" $\Lambda_2(\boxed{62})$	
			with $I=4$ as subshift of time-1 Bernoulli $\sigma_{ au}$ -shift map	
			(with $\sigma = 1$) $\phi_n = 2\phi_{n-1} \mod \nu(I)$ for rule 170	889

	7.2.6. Analytical proof of time-2 map of $\Lambda_2(\boxed{62})$ with	
	$I=4$ is a subshift of time-2 Bernoulli $\sigma_{ au}$ -shift map	
	(with $\sigma = -1$) of Eq. (85) of rule $\boxed{240}$	892
	7.2.7. Analytical proof of time-1 map of $\Lambda_4(\boxed{62})$ with	
	$I=4$ is a subshift of time-1 Bernoulli $\sigma_{ au}$ -shift map	
	(with $\sigma = 2$) of Eq. (63) of rule 170	893
	7.3. $\Lambda_4(62)$ is a subshift of $\Lambda(240)$	895
8.	The Arrow of Time	899
	8.1. Rule 6	899
	8.2. Rule 9	899
	8.3. Rule 25	899
	8.4. Rule 74	910
9.	Concluding Remarks	910
	9.1. Attractors of 206 local rules	910
	9.2. Time reversality	911
	9.3. Paradigm shift via nonlinear dynamics	913
Errata for Volume I		
Epilogue		
References		
Index (for Volumes I and II)		