

## CONTENTS

|   |    |
|---|----|
| Preface.....  | ii |
| Chapter 1. Basic Facts about Holomorphic Functions                                      |    |
| §1. Preliminaries.....  | 1  |
| §2. An inequality.....  | 5  |
| §3. Proof of Hartogs' Theorem 1.....  | 7  |
| §4. Holomorphic mappings.....   | 10 |
| Chapter 2. Domains of Holomorphy  |    |
| §1. Examples and definitions.....   | 12 |
| §2. Convexity with respect to a family of functions.....                                | 14 |
| §3. Domains of convergence of power series.....   | 18 |
| §4. Bergman domains.....  | 22 |
| §5. Analytic polyhedra.....   | 24 |
| Chapter 3. Pseudoconvexity  |    |
| §1. Plurisubharmonic and pseudoconvex functions.....                                    | 26 |
| §2. Pseudoconvex domains.....   | 30 |
| §3. Solution of the Levi Problem for tube domains.....                                  | 35 |
| Chapter 4. Zeros of Holomorphic Functions.  |    |
| Meromorphic Functions.  |    |
| §1. Weierstrass Preparation Theorem.....  | 38 |
| §2. Rings of power series.....  | 44 |
| §3. Meromorphic functions.....  | 48 |
| §4. Removable singularities.....  | 50 |
| §5. Complex manifolds.....  | 53 |
| Chapter 5. The Additive Cousin Problem  |    |
| §1. The additive Problem formulated.....  | 59 |
| §2. Reformulation of the Cousin Problem.....  | 61 |
| §3. Reduction of the Cousin Problem to<br>non-homogeneous Cauchy-Riemann equations..... | 63 |
| Chapter 6. Cohomology   |    |
| §1. Cohomology of a complex manifold with<br>holomorphic functions as coefficients..... | 69 |
| §2. Applications.....   | 74 |
| §3. Other cohomologies.....   | 76 |
| Chapter 7. Differential Forms   |    |
| §1. Ring of differential forms in a domain.....   | 80 |
| §2. Differential forms on manifolds.....  | 84 |
| §3. Poincaré Lemmas.....  | 85 |
| Chapter 8. Canonical Isomorphisms   |    |
| §1. De Rham's Theorem.....  | 89 |
| §2. Dolbeault's Theorem.....  | 93 |
| §3. Complex de Rham Theorem.....  | 96 |

## Chapter 9. The Multiplicative Cousin Problem

|   |     |
|---|-----|
| §1. The Multiplicative Problem, formulated.....                             | 98  |
| §2. The Multiplicative Cousin Problem is not<br>always solvable.....        | 100 |
| §3. The solution of the Multiplicative Cousin<br>Problem for polydiscs..... | 103 |
| §4. Characteristic classes (From C.II to C.I).....                          | 106 |

## Chapter 10. Runge Regions

|                               |     |
|-------------------------------|-----|
| §1. Preliminaries.....        | 110 |
| §2. Polynomial polyhedra..... | 112 |
| §3. Runge domains.....        | 113 |

## Chapter 11. Cohomology of Domains of Holomorphy

|  |     |
|--|-----|
| §1. Fundamental Lemma, stated.....                             | 115 |
| §2. Applications of the Fundamental Lemma.....                 | 115 |
| §3. Preparation for the proof of the<br>Fundamental Lemma..... | 117 |
| §4. Proof of the Fundamental Lemma.....                        | 120 |

## Chapter 12. Some Consequences of the Approximation Theorem

|   |     |
|---|-----|
| §1. Relative convexity.....               | 128 |
| §2. Unbounded regions of holomorphy.....  | 129 |
| §3. The Behnke-Stein Theorem.....         | 130 |
| §4. Applications to the Levi Problem..... | 132 |

## Chapter 13. Solution of the Levi Problem

|  |     |
|--|-----|
| §1. Reduction to a finiteness statement..... | 134 |
| §2. Reduction to an extension property.....  | 137 |
| §3. Proof of Proposition 2.....              | 140 |

## Chapter 14. Sheaves

|   |     |
|---|-----|
| §1. Exact sequences.....  | 142 |
| §2. Differential operators.....                                   | 144 |
| §3. Graded groups.....  | 147 |
| §4. Sheaves and pre-sheaves.....                                  | 149 |
| §5. Exact sequences of sheaves and cohomology.....                | 150 |
| §6. Applications of the exact cohomology<br>sequence theorem..... | 155 |
| §7. Proof of the exact cohomology sequence theorem....            | 158 |

## Chapter 15. Coherent Analytic Sheaves

|   |     |
|---|-----|
| §1. Definitions.....                                | 162 |
| §2. Oka's coherence theorem.....                    | 163 |
| §3. Weierstrass Preparation Theorem, revisited..... | 165 |
| §4. The third step.....                             | 168 |
| §5. Consequences of Oka's theorem.....              | 171 |
| §6. The sheaf of ideals of a variety.....           | 173 |

**Chapter 16. Fundamental Theorems (semi-local form)**

|   |     |
|---|-----|
| §1. Statement of the fundamental theorems for<br>a box (semi-local form)..... | 175 |
| §2. First step of the proof.....  | 175 |
| §3. Reduction of (3) to Cartan's theorem<br>on holomorphic matrices.....      | 177 |
| §4. Proof of Cartan's theorem on holomorphic matrices.                        | 180 |
| §5. New proof of the Oka-Weil Approximation Theorem...                        | 184 |
| §6. Fundamental theorems for regions of<br>holomorphy (semi-local form).....  | 185 |

**Chapter 17. Coherent Sheaves in Regions of Holomorphy**

|   |     |
|---|-----|
| §1. Statement of the fundamental theorems.....    | 187 |
| §2. Preparations for the proof.....               | 187 |
| §3. Proof of Theorem A.....                       | 191 |
| §4. Proof of Theorem B.....                       | 194 |
| §5. Applications of the fundamental theorems..... | 196 |

**Chapter 18. Stein Manifolds (Holomorphically  
Complete Manifolds)**

|   |     |
|---|-----|
| §1. Definition and examples.....                      | 201 |
| §2. An approximation theorem.....                     | 202 |
| §3. The fundamental theorems for Stein manifolds..... | 203 |
| §4. Characterization of Stein manifolds.....          | 203 |

|               |     |
|---------------|-----|
| Appendix..... | 205 |
|---------------|-----|