Contents

	Preface	page	хi
1	Groups of matrices and bilinear mappings		
1.1	Notation		1
1.2	The modular group		7
1.3	The subgroups Γ^2 , Γ^3 , Γ^4 and $\Gamma'(1)$		15
1.4	The level of a subgroup; congruence subgroups	•	19
1.5	Groups of level 2		28
1.6	Groups of level 3		30
1.7	Further results		32
2	Mapping properties		
2.1	Conformal mappings		39
2.2	Fixed points		42
2.3	Fundamental regions		47
2.4	Construction of fundamental regions for $\hat{\Gamma}(1)$		
	and its subgroups		51
2.5	Further results		66
3	Automorphic factors and multiplier systems		
3.1	Introduction		70
3.2	The functions $\sigma(S, T)$ and $w(S, T)$		73
3.3	Automorphic factors on subgroups of the		
	modular group		77
3.4	Multiplier systems on $\Gamma(1)$, Γ^2 and Γ^3		82
3.5	Further results		86
4	General properties of modular forms		
4.1	Definitions and general theorems		88
4.2	Dimensions of spaces of modular forms	1	02
4.3	Relations between modular forms	1	.08
4.4	Modular forms of weight 2	1	22
4.5	Orders of magnitude	1	28
4.6	Further results	1	30

viii Contents

5	Construction of modular forms	
5.1	Poincaré series	134
5.2	The Hilbert space of cusp forms	144
5.3	The Fourier coefficients of Poincaré series	155
5.4	Kloosterman sums	164
5.5	Poincaré series belonging to $\Gamma(N)$	172
5.6	Poincaré series on $\Gamma(N^2)$	179
5.7	Modular forms on $\bar{\Gamma}(N)$ of weight 2	183
5.8	Further results	191
6	Functions belonging to the full modular group	
6.1	Modular forms of even weight with constant	
	multiplier system	194
6.2	Poincaré series	201
6.3	The case $\delta_k = 1$	202
6.4	Modular forms of any real weight	205
6.5	Modular equations	209
6.6	Further remarks	212
7	Groups of level 2 and sums of squares	
7.1	Hermite and theta functions	215
7.2	Modular functions of level 2	226
7.3	Eisenstein series belonging to $\Gamma(2)$	232
7.4	Functions belonging to $\Gamma_V(2)$; sums of squares	238
7.5	Further remarks	243
8	Modular forms of level N	
8.1	Forms of fixed character or divisor	245
8.2	The interaction of the operators R^{x} and D_{t}	250
8.3	The operator J_n	254
8.4	The operator H_q	260
8.5	The operator L_q	264
8.6	The conjugate linear map K	269
8.7	Historical remarks	27 1
9	Hecke operators and congruence groups	
9.1	Double coset modules	273
9.2	Definition and properties of Hecke operators	289
0.2	The effect of Heelra apprehens on Doingaré series	301

Contents

ix

9.4	Eigenforms	317
9.5	Historical and other remarks	333
10	Applications	
10.1	Dirichlet series	337
10.2	Eigenforms for the full modular group	342
10.3	Eigenforms for $\Gamma_V(2)$ and $\Gamma(2)$	343
10.4	Eigenforms of level 4	350
10.5	Final remarks	359
	Bibliography	361
	Index of special symbols	369
	Index of authors	380
	Subject index	382