

Contents

CHAPTER 1.	INTRODUCTION	1
CHAPTER 2.	DIFFERENTIATION OF INTEGRALS	6
CHAPTER 3.	CONSISTENCY	12
	1. Kernel Estimate, 12	
	2. Proof of Theorem 1, 13	
	3. Histogram Estimate, 19	
	4. Proof of Theorem 2, 21	
	5. Relative Stability, 23	
CHAPTER 4.	LOWER BOUNDS FOR RATES OF CONVERGENCE	35
	1. Introduction, 35	
	2. Assouad's Lemma, 40	
	3. Some Historical Remarks, 46	
	4. Proof of Theorem 1, 50	
	5. Proof of Theorems 2 and 11, 57	
	6. Proof of Theorem 3, 58	
	7. Proof of Theorem 4, 62	
	8. Proof of Theorems 5, 6, 7, 8, and 9, 64	
CHAPTER 5.	RATES OF CONVERGENCE IN L_1	76
	1. Introduction, 76	
	2. The Factor $B^*(f)$, 80	
	3. Proofs of Theorems 1 and 2, 89	
	4. The Histogram Estimate, 97	

5. Proofs of Theorems 5 and 6, 100
6. Choice of the Smoothing Parameter, 107
7. The Uniform Density, 113
8. A Minimax Strategy for Choosing the Smoothing Factor, 117
9. Lipschitz Classes, Bretagnolle–Huber Classes, and Uniform Upper Bounds, 121
10. Densities with Unbounded Support, 129
11. Unbiasedness and the Achievability of the Error Rate $1/\sqrt{n}$, 133

**CHAPTER 6. THE AUTOMATIC KERNEL ESTIMATE:
 L_1 AND POINTWISE CONVERGENCE** 148

1. The Main Result, 148
2. Pointwise Convergence of the Automatic Kernel Estimate, 148
3. Pointwise Convergence of the Standard Kernel Estimate, 149
4. Examples of Automatic Kernel Estimates, 150
5. Proofs, 156
6. Invariate Density Estimation, 183
7. Rate of Convergence for Automatic Kernel Estimates, 186

**CHAPTER 7. ESTIMATES RELATED TO THE
KERNEL ESTIMATE AND
THE HISTOGRAM ESTIMATE** 191

1. Introduction, 191
2. Variable Kernel Estimates, 192
3. Recursive Kernel Estimates, 193
4. Maximum Likelihood Estimates, 201
5. Variable Histogram Estimates, 204
6. Kernel Estimates with Reduced Bias, 205
7. Grenander's Estimate for Monotone Densities, 213

**CHAPTER 8. SIMULATION, INEQUALITIES, AND
RANDOM VARIATE GENERATION** 220

1. Choosing a Criterion, 220
2. Inequalities, 221
3. The Generalization of a Sample for Random Variate Generation, 227

CHAPTER 9. THE TRANSFORMED KERNEL ESTIMATE	244
1. Introduction, 244	
2. Choosing a Transformation, 246	
3. Estimation of Densities with Large Tails, 247	
4. Consistency, 250	
CHAPTER 10. APPLICATIONS IN DISCRIMINATION	253
1. The Discrimination Problem, 253	
2. Slow Rates of Convergence, 255	
3. The Kernel Method in Discrimination, 257	
4. Histogram-Based Discrimination, 258	
5. The Nearest-Neighbor Method, 259	
CHAPTER 11. OPERATIONS ON DENSITY ESTIMATES	267
1. Marginal Densities, 267	
2. Composition (Mixtures) of Densities, 268	
3. Restrictions of Densities, 269	
4. Nonnegative Projections, 269	
5. Product Densities, 270	
6. Radially Symmetric Densities, 271	
7. Convolutions, 272	
8. Unimodal Densities, 273	
9. Applications in Detection, 274	
10. Symmetrization and Permutation Invariance, 281	
CHAPTER 12. ESTIMATORS BASED ON ORTHOGONAL SERIES	286
1. Definitions, 286	
2. Examples of Orthonormal Systems, 289	
3. General Properties, 292	
4. The Trigonometric Series Estimate: Consistency, 294	
5. The Trigonometric Series Estimate: Rate of Convergence, 304	
6. The Hermite Series Estimate, 312	
7. The Legendre Series Estimate, 316	
8. Singular Integral Estimates, 319	
AUTHOR INDEX	343
SUBJECT INDEX	347