

TABLE OF CONTENTS

TABLE OF CONTENTS.

PREFACE.

I. DISTANCES BETWEEN DENSITIES.	1
1.1. The total variation.	1
1.2. The space L_p .	4
1.3. Hellinger spaces.	7
1.4. Entropy. Kullback-Leibler numbers.	9
1.5. Standard improvements of density estimates.	12
1.6. Projection estimates.	14
1.7. Exercises.	15
II. DENSITY ESTIMATION AND DERIVATION OF MEASURES.	17
2.1. Our model.	17
2.2. Popular density estimates.	17
2.3. Differentiation of integrals.	22
2.4. Characteristic functions.	24
2.5. Integral convergence from pointwise convergence.	25
2.6. Exercises.	28
III. CONSISTENCY OF THE KERNEL ESTIMATE.	30
3.1. The equivalence theorem.	30
3.2. Simple splits into bias and variation.	31
3.3. A large deviation inequality for the multinomial distribution.	31
3.4. Proof of (E) \Rightarrow (D).	32
3.5. Proof of (A) \Rightarrow (E).	37
3.6. Data-based smoothing.	38
3.7. Exercises.	42
IV. ROBUSTNESS.	43
4.1. Definition.	43
4.2. An example: a parametric estimate.	45
4.3. The kernel estimate.	46
4.4. Application: Beran's robust parametric estimates.	49

4.5. Exercises.	50
V. MINIMAX BOUNDS.	51
5.1. Minimax theory.	51
5.2. The low-probability method.	53
5.3. Examples of rich classes.	54
5.4. Information-theoretic methods.	59
5.5. A centered class.	62
5.6. A Lipschitz class.	65
5.7. Mixture classes.	70
5.8. Convolution classes.	72
5.9. Fano's lemma.	75
5.10. Lower bounds via sufficient statistics.	81
5.11. Construction of good minimax estimators.	85
5.12. Exercises.	85
VI. MINIMUM DISTANCE ESTIMATORS.	88
6.1. Definition.	88
6.2. The key inequality.	90
6.3. Construction of an ϵ -cover.	94
6.4. Kolmogorov's entropy.	97
6.5. Exercises.	98
VII. RATE OF CONVERGENCE OF KERNEL ESTIMATES.	99
7.1. Scope of this chapter.	99
7.2. Classes of kernels.	100
7.3. Universal derivatives and mollifiers.	106
7.4. The bias of the kernel estimate for class s kernels.	107
7.5. Saturation and unbiasedness.	112
7.6. The variation of the kernel estimate.	113
7.7. Minimax upper bounds.	117
7.8. The optimal kernel.	119
7.9. Individual upper bounds.	122
7.10. Modified kernel estimates.	126
7.11. Rates of convergence with superkernels.	127
7.12. Exercises.	129
VIII. A CASE STUDY: MONOTONE DENSITIES ON $[0,1]$.	133
8.1. Scope of this chapter.	133
8.2. The minimax lower bound.	134
8.3. Grenander's estimate.	138
8.4. The kernel estimate.	147
8.5. Birge's modified histogram estimate.	151
8.6. Exercises.	157
IX. RELATIVE STABILITY.	160
9.1. Definition and motivation.	160
9.2. Main results.	161
9.3. A moment inequality for the Poisson distribution.	164
9.4. Two fundamental tools.	166
9.5. Proof of Theorem 9.1.	170

TABLE OF CONTENTS

vii

9.6. Exercises.	173
REFERENCES.	175
INDEX.	180