Contents

Preface v

CHAPTER I
Introduction and Examples 1

CHAPTER II
The Sequential Probability Ratio Test 8

1. Definition and Examples 8
2. Approximations for $P_i\{l_N \geq B\}$ and $E_i(N)$ 10
3. Tests of Composite Hypotheses 14
4. Optimality of the Sequential Probability Ratio Test 19
5. Criticism of the Sequential Probability Ratio Test and the Anscombe–Doeblin Theorem 22
6. Cusum Procedures 24

CHAPTER III
Brownian Approximations and Truncated Tests 34

1. Introduction 34
2. Sequential Probability Ratio Test for the Drift of Brownian Motion 36
3. Truncated Sequential Tests 37
4. Attained Significance Level and Confidence Intervals 43
5. Group Sequential Tests and the Accuracy of Brownian Approximations 49
6. Truncated Sequential Probability Ratio Test 51
7. Anderson's Modification of the Sequential Probability Ratio Test 58
8. Early Stopping to Accept H_0 62
9. Brownian Approximation with Nuisance Parameters 63

CHAPTER IV
Tests with Curved Stopping Boundaries 70

1. Introduction and Examples 70
2. Repeated Significance Tests for Brownian Motion 73
3. Numerical Examples for Repeated Significance Tests 81
4. Modified Repeated Significance Tests 86
5. Attained Significance Level and Confidence Intervals 89
6. Discussion 93
7. Some Exact Results 95
8. The Significance Level of Repeated Significance Tests for General One-Parameter Families of Distributions 98

CHAPTER V
Examples of Repeated Significance Tests 105

1. Introduction 105
2. Bernoulli Data and Applications 106
3. Comparing More than Two Treatments 111
4. Normal Data with Unknown Variance 116
5. Survival Analysis—Theory 121
6. Survival Analysis—Examples and Applications 129

CHAPTER VI
Allocation of Treatments 141

1. Randomization Tests 141
2. Forcing Balanced Allocation 144
3. Data Dependent Allocation Rules 148
4. Loss Function and Allocation 150

CHAPTER VII
Interval Estimation of Prescribed Accuracy 155

1. Introduction and Heuristic Stopping Rule 155
2. Example—The Normal Mean 156
3. Example—The Log Odds Ratio 159

CHAPTER VIII
Random Walk and Renewal Theory 165

1. The Problem of Excess over a Boundary 165
2. Reduction to a Problem of Renewal Theory and Ladder Variables 167
3. Renewal Theory 168
4. Ladder Variables 172
5. Applications to Sequential Probability Ratio Tests and Cusum Tests 179
6. Conditioned Random Walks 181

CHAPTER IX
Nonlinear Renewal Theory 188

1. Introduction and Examples 188
2. General Theorems 189
3. Applications to Repeated Significance Tests 198
4. Application to Fixed Width Confidence Intervals for a Normal Mean 207
5. Woodroofe's Method 208

CHAPTER X
Corrected Brownian Approximations 213

1. $P_{\mu_0}\{\tau(b) < \infty\}$ Revisited 213
2. Sequential Probability Ratio Tests and Cusum Tests 216
3. Truncated Tests 220
4. Computation of $E_0(S^2_{\tau_-})/2E_0(S_{\tau_-})$ 224

CHAPTER XI
Miscellaneous Boundary Crossing Problems 229

1. Proof of Theorem 4.21 229
2. Expected Sample Size in the Case of More than Two Treatments 232
3. The Discrete Brownian Bridge 234

APPENDIX 1
Brownian Motion 241

APPENDIX 2
Queueing and Insurance Risk Theory 245

APPENDIX 3
Martingales and Stochastic Integrals 248

APPENDIX 4
Renewal Theory 253

Bibliographical Notes 258

References 263

Index 271