Contents

1 The Finite Element Method 1

1.1 Fundamental Concept of the Finite Element Method 2
1.2 Advantages and Disadvantages 7
1.3 Scope 8

2 Discretization of the Domain 10

2.1 Types of Elements 11
2.2 The Division of the Domain into Elements 14
2.3 Labeling of the Nodes 18
2.4 Summary 19

3 Linear Interpolation Polynomials 23

3.1 One-Dimensional Simplex Element 24
3.2 Two-Dimensional Simplex Element 28
3.3 Three-Dimensional Simplex Element 33
3.4 Interpolation for Vector Quantities 37
3.5 Local Coordinate Systems 39
3.6 Properties of the Interpolation Polynomial 46

4 *Interpolating Polynomials for a Discretized Region* 54

4.1 Scalar Quantities 55
4.2 Vector Quantities 58
4.3 Summary 60

5 *A Finite Element Formulation for Some Boundary Value Problems* 63

5.1 A Simple Example: Heat Conduction in a Rod 64
5.2 The Example Problem Reconsidered 69
5.3 Finite Element Equations: Field Problems 71
5.4 Finite Element Equations: Theory of Elasticity 77

6 *Torsion of Noncircular Sections* 87

6.1 General Theory, Torsion of Noncircular Sections 88
6.2 Assemblage of the Element Matrices 90
6.3 Conventional Element Resultants 96
6.4 Consistent Element Resultants 100

7 *Finite Element Methodology: Computer Implementation* 105

7.1 Direct Construction of the Global Stiffness Matrix 105
7.2 The System of Linear Equations 109
7.3 General Computer Flow Diagram 116
7.4 Computer Implementation of the Torsion Problem 122

8 *Heat Transfer by Conduction and Convection* 138

8.1 Heat Transfer Equations 139
8.2 One-Dimensional Heat Transfer 141
8.3 Two-Dimensional Heat Transfer 148
8.4 Three-Dimensional Heat Transfer 155
8.5 Coordinate Transformations 157

8.6 Point Sources 157
8.7 Computer Implementation 162

9 Fluid Mechanics: Irrotational Flow 174

9.1 Two-Dimensional Groundwater Flow 175
9.2 Computer Implementation of Groundwater Problems 176
9.3 Irrotational Flow of an Ideal Fluid 182
9.4 Summary 184

10 Radial and Axisymmetric Field Problems 191

10.1 Two-Dimensional Symmetric Field Problems 192
10.2 Axisymmetric Field Problems 199
10.3 Computer Implementation 208

11 Time Dependent Field Problems 212

11.1 The Element Equations 212
11.2 The Element Capacitance Matrix 215
11.3 Finite Difference Solution in the Time Domain 218
11.4 Numerical Stability and Oscillations 221
11.5 Computer Implementation 222

12 Solid Mechanics: Elasticity 224

12.1 One-Dimensional Elasticity 225
12.2 Two-Dimensional Elasticity 232
12.3 Three-Dimensional Elasticity 240
12.4 Axisymmetric Elasticity 243
12.5 Computer Implementation 248

13 Higher-Order Elements: The One-Dimensional Element 257

13.1 The Quadratic and Cubic Elements 258
13.2 Application of the Quadratic Element 263

13.3 Natural Coordinate System. Coordinate Equations. Jacobian Matrix 269
13.4 Numerical Integration to Determine the Element Matrices 274
13.5 Subparametric, Isoparametric, and Superparametric Elements 280

14 Higher-Order Triangular and Tetrahedral Elements 287

14.1 Shape Functions for Higher-Order Elements 289
14.2 Calculation of the Derivatives of the Shape Functions 293
14.3 Evaluation of the Element Matrices 297
14.4 Tetrahedral Elements 303

15 The Quadrilateral Elements 309

15.1 The Linear Quadrilateral Element 309
15.2 Quadratic and Cubic Quadrilateral Elements 314
15.3 Evaluation of the Shape Function Derivatives 321
15.4 Evaluation of the Element Equations 325
15.5 Rectangular Prisms 329

16 Higher-Order Elements: Computer Implementation 334

16.1 Computer Implementation 334
16.2 Application Examples 337
16.3 Curved Boundaries 342

17 Formulation of the Element Equations Using Galerkin's Method 345

17.1 Galerkin's Method 346
17.2 Beam Deflections 347

17.3 Two-Dimensional Field Equation 352
17.4 Initial Value Problems 355
17.5 A System of First-Order Differential Equations 360
17.6 Summary 362

18 Computer Programs for Finite Element Instruction 365

18.1 GRID 367
18.2 Band Matrix Subroutines 377
18.3 Notation 379
18.4 Torsion Program 381
18.5 CONSTR 382
18.6 FLDMCH 387
18.7 TDHEAT 387
18.8 STRESS 391

19 Concluding Remarks 400

APPENDIX A Some Aspects of the Calculus of Variations 402

APPENDIX B Differentiation of Matrix Equations 406

Answers to Selected Problems 409
Index 417