
Contents

Preface	xi
1 Introduction	1
1.1 Rank data	1
2 Looking at Data	5
2.1 Introduction: Permutation polytopes	5
2.2 Projections of polytopes	12
2.3 Marginals	17
2.4 Pairs	19
2.5 Center, spread, and distance	21
2.5.1 Some useful distances	23
2.5.2 Estimating the center	28
2.5.3 Estimating spread, location known	30
2.5.4 Estimating spread, location unknown	31
2.5.5 Clustering: L -centers	33
2.6 Linear subspaces	39
2.6.1 Spectral decomposition	45
2.6.2 Inversions	52
2.7 Exercises	54
3 Formal Tests of Uniformity	56
3.1 Introduction	56
3.2 Summary statistics	57
3.2.1 Projections	58
3.2.2 Probabilities	60
3.2.3 Marginals	61
3.2.4 Means	63
3.2.5 Pairs	64
3.3 Invariance and monotonicity of distances	68
3.4 Distance from a fixed vector	73
3.4.1 Means and variances	75

3.4.2	Exact distributions	77
3.4.3	Asymptotics as $m \rightarrow \infty$	82
3.5	One-sample diversity and concordance	86
3.6	Summary	92
3.7	Exercises	93
4	Comparing Populations of Judges	97
4.1	Introduction	97
4.2	The basic statistics:	
	Multivariate analysis of variance tests	98
4.3	Distances from a modal ranking	101
4.4	Concordance and diversity	104
4.5	Exercises	110
5	Overview of Models	111
5.1	Introduction	111
5.2	Probability models — General	112
5.3	Thurstonian \equiv Order statistic models	114
5.4	Distance-based models	115
5.5	Paired comparison models — Babington Smith	115
	5.5.1 Bradley–Terry/Mallows	117
	5.5.2 Mallows' models	118
5.6	Multistage models	118
	5.6.1 Plackett–Luce	118
	5.6.2 Free and ϕ -component models	120
5.7	Sufficient statistic models	122
5.8	Loglinear models	123
5.9	ANOVA-like models	124
5.10	Nested orthogonal contrast models	126
	5.10.1 The free model	129
	5.10.2 The ϕ model	129
	5.10.3 Contingency table models	130
5.11	Unfolding models	132
5.12	Generalizing the models	133
5.13	Some axiomatics	134
	5.13.1 Luce's choice axiom	134
	5.13.2 Unidimensionality, unimodality, and consensus	136
5.14	Likelihood methods and exponential families	140
	5.14.1 The likelihood function and Fisher information	141
	5.14.2 Maximum likelihood estimation	143

5.14.3	Likelihood ratio tests	143
5.14.4	Exponential families	145
5.15	Exercises	147
6	Distance-Based Models	149
6.1	Introduction	149
6.2	Fitting the models: Known mode	149
6.2.1	Kendall	151
6.2.2	Cayley	151
6.2.3	Hamming	152
6.3	Unknown mode – Likelihood	154
6.4	Unknown mode – Bayesian	156
6.5	Asymptotics as $m \rightarrow \infty$	158
6.5.1	Kendall	159
6.5.2	Cayley	160
6.5.3	Hamming	161
6.5.4	Maximum	161
6.6	Assessing fit	162
6.6.1	Kendall	163
6.6.2	Hamming and Cayley	165
6.7	Exercises	168
7	Babington Smith, Phi-Models, and Inversions	170
7.1	Introduction	170
7.2	Babington Smith	172
7.3	Contrast ϕ models	175
7.4	Bradley–Terry/Mallows and Spearman’s distance models	178
7.4.1	Bradley–Terry/Mallows	179
7.4.2	Submodels of Bradley–Terry/Mallows, including Spearman	180
7.4.3	Between Babington Smith and Bradley–Terry/Mallows	180
7.5	Basic results for orthogonal contrast ϕ models	182
7.6	Details for the orthogonal contrast ϕ models	184
7.6.1	Preliminaries	185
7.6.2	Null distribution for <i>Kendall</i>	187
7.6.3	Non-null distribution for <i>Phi</i> , <i>Mallows</i> , and <i>Kendall</i>	189
7.6.4	Decomposing Mallows’ ϕ model; Proofs of Theorems 6.3 and 6.5	191

7.7	Examples of orthogonal contrast models	193
7.8	Mixed orthogonal contrast/Babington Smith models	198
7.9	Multistage models and patterns of ties	200
7.9.1	Patterns of ties	200
7.9.2	Mallows' ϕ model for tied rankings	202
7.9.3	Multistage models: ϕ -component	203
7.10	Free models and contingency tables	204
7.11	Inversion models	209
7.12	Exercises	211
8	Plackett-Luce, Logistic, and Vase Models	213
8.1	The first-order model	213
8.2	Extended vase models: No interaction	217
8.3	Extended Plackett models	219
8.4	q -permutations	226
9	Marginal and ANOVA-Type Loglinear Models	228
9.1	Introduction	228
9.2	Submodels of the Marginals model	228
9.3	Extended Marginals models	232
9.4	ANOVA-type loglinear models	234
9.5	Paired and multisample models	236
10	Latent Class and Unfolding Models	243
10.1	Introduction	243
10.2	Latent class models	244
10.3	The EM algorithm for latent class models	247
10.4	Unidimensional unfolding models	251
10.5	Exercise	258
11	Tied, Partial, and Incomplete Rankings	259
11.1	Introduction	259
11.1.1	Tied rankings	261
11.1.2	Rankings of subsets:	
	Balanced incomplete block designs	261
11.2	Descriptive statistics	263
11.3	Tests of uniformity	265
11.4	Tied rankings	269
11.4.1	Marginals and Means	270
11.4.2	Pairs	271

11.4.3 Distances	273
11.5 Rankings of subsets:	
Balanced incomplete block designs	275
11.5.1 Marginals and Means; Hamming's and Spearman's distances	275
11.5.2 Pairs and Kendall's distance	278
11.6 Comparing samples	280
11.7 Models	281
11.7.1 Thurstonian models	283
11.7.2 Plackett-Luce and vase models	284
11.7.3 Babington Smith	284
11.7.4 Orthogonal contrast ϕ models: Tied rankings	286
11.7.5 Orthogonal contrast ϕ models: Subsets	287
11.7.6 Exponential family models	288
11.7.7 Coefficients and comparing populations	289
11.8 Exercises	291
12 Appendix	294
12.1 Some linear algebra	294
12.1.1 Eigenvalues and eigenvectors	295
12.2 Means and covariances for vectors and matrices	296
12.2.1 Definitions	296
12.2.2 Kronecker products	297
12.3 Normality and chi-squares	299
12.4 Some asymptotics	300
12.4.1 Central limit theorem	300
12.4.2 Convergence in probability	301
Bibliography	302
Author Index	311
Subject Index	314