Contents

CHAPTER 1
Basic Concepts 1

1.1 Introduction 1
1.2 Basic Elements 3
1.3 Expected Loss, Decision Rules, and Risk 8
1.3.1 Bayesian Expected Loss 8
1.3.2 Frequentist Risk 9
1.4 Randomized Decision Rules 12
1.5 Decision Principles 16
1.5.1 The Conditional Bayes Decision Principle 16
1.5.2 Frequentist Decision Principles 16
1.6 Foundations 20
1.6.1 Misuse of Classical Inference Procedures 20
1.6.2 The Frequentist Perspective 22
1.6.3 The Conditional Perspective 24
1.6.4 The Likelihood Principle 27
1.6.5 Choosing a Paradigm or Decision Principle 33
1.7 Sufficient Statistics 35
1.8 Convexity 38
Exercises 41

CHAPTER 2
Utility and Loss 46

2.1 Introduction 46
2.2 Utility Theory 47
2.3 The Utility of Money 53
2.4 The Loss Function 57
2.4.1 Development from Utility Theory 57

2.4.2 Certain Standard Loss Functions 60
2.4.3 For Inference Problems 64
2.4.4 For Predictive Problems 66
2.4.5 Vector Valued Loss Functions 68
2.5 Criticisms 69
Exercises 70

CHAPTER 3
Prior Information and Subjective Probability 74

3.1 Subjective Probability 74
3.2 Subjective Determination of the Prior Density 77
3.3 Noninformative Priors 82
3.3.1 Introduction 82
3.3.2 Noninformative Priors for Location and Scale Problems 83
3.3.3 Noninformative Priors in General Settings 87
3.3.4 Discussion 89
3.4 Maximum Entropy Priors 90
3.5 Using the Marginal Distribution to Determine the Prior 94
3.5.1 The Marginal Distribution 94
3.5.2 Information About m 95
3.5.3 Restricted Classes of Priors 97
3.5.4 The ML-II Approach to Prior Selection 99
3.5.5 The Moment Approach to Prior Selection 101
3.5.6 The Distance Approach to Prior Selection 103
3.5.7 Marginal Exchangeability 104
3.6 Hierarchical Priors 106
3.7 Criticisms 109
3.8 The Statistician's Role 113
Exercises 113

CHAPTER 4
Bayesian Analysis 118

4.1 Introduction 118
4.2 The Posterior Distribution 126
4.2.1 Definition and Determination 126
4.2.2 Conjugate Families 130
4.2.3 Improper Priors 132
4.3 Bayesian Inference 132
4.3.1 Estimation 133
4.3.2 Credible Sets 140
4.3.3 Hypothesis Testing 145
4.3.4 Predictive Inference 157
4.4 Bayesian Decision Theory 158
4.4.1 Posterior Decision Analysis 158
4.4.2 Estimation 161
4.4.3 Finite Action Problems and Hypothesis Testing 163
4.4.4 With Inference Losses 166

4.5 Empirical Bayes Analysis 167
4.5.1 Introduction 167
4.5.2 PEB For Normal Means—The Exchangeable Case 169
4.5.3 PEB For Normal Means—The General Case 173
4.5.4 Nonparametric Empirical Bayes Analysis 178
4.6 Hierarchical Bayes Analysis 180
4.6.1 Introduction 180
4.6.2 For Normal Means—The Exchangeable Case 183
4.6.3 For Normal Means—The General Case 190
4.6.4 Comparison with Empirical Bayes Analysis 193
4.7 Bayesian Robustness 195
4.7.1 Introduction 195
4.7.2 The Role of the Marginal Distribution 199
4.7.3 Posterior Robustness: Basic Concepts 203
4.7.4 Posterior Robustness: ε-Contamination Class 206
4.7.5 Bayes Risk Robustness and Use of Frequentist Measures 213
4.7.6 Gamma-Minimax Approach 215
4.7.7 Uses of the Risk Function 218
4.7.8 Some Robust and Nonrobust Situations 223
4.7.9 Robust Priors 228
4.7.10 Robust Priors for Normal Means 236
4.7.11 Other Issues in Robustness 247
4.8 Admissibility of Bayes Rules and Long Run Evaluations 253
4.8.1 Admissibility of Bayes Rules 253
4.8.2 Admissibility of Generalized Bayes Rules 254
4.8.3 Inadmissibility and Long Run Evaluations 257
4.9 Bayesian Calculation 262
4.9.1 Numerical Integration 262
4.9.2 Monte Carlo Integration 263
4.9.3 Analytic Approximations 265
4.10 Bayesian Communication 267
4.10.1 Introduction 267
4.10.2 An Illustration: Testing a Point Null Hypothesis 268
4.11 Combining Evidence and Group Decisions 271
4.11.1 Combining Probabilistic Evidence 272
4.11.2 Combining Decision-Theoretic Evidence 277
4.11.3 Group Decision Making 278
4.12 Criticisms 281
4.12.1 Non-Bayesian Criticisms 281
4.12.2 Foundational Criticisms 283
Exercises 286

CHAPTER 5
Minimax Analysis 308

5.1 Introduction 308
5.2 Game Theory 310
5.2.1 Basic Elements 310
5.2.2 General Techniques for Solving Games 319

5.2.3 Finite Games 325
5.2.4 Games with Finite Θ 331
5.2.5 The Supporting and Separating Hyperplane Theorems 339
5.2.6 The Minimax Theorem 345
5.3 Statistical Games 347
5.3.1 Introduction 347
5.3.2 General Techniques for Solving Statistical Games 349
5.3.3 Statistical Games with Finite Θ 354
5.4 Classes of Minimax Estimators 359
5.4.1 Introduction 359
5.4.2 The Unbiased Estimator of Risk 361
5.4.3 Minimax Estimators of a Normal Mean Vector 363
5.4.4 Minimax Estimators of Poisson Means 369
5.5 Evaluation of the Minimax Principle 370
5.5.1 Admissibility of Minimax Rules 371
5.5.2 Rationality and the Minimax Principle 371
5.5.3 Comparison with the Bayesian Approach 373
5.5.4 The Desire to Act Conservatively 376
5.5.5 Minimax Regret 376
5.5.6 Conclusions 378
Exercises 379

CHAPTER 6
Invariance 388

6.1 Introduction 388
6.2 Formulation 391
6.2.1 Groups of Transformations 391
6.2.2 Invariant Decision Problems 393
6.2.3 Invariant Decision Rules 395
6.3 Location Parameter Problems 397
6.4 Other Examples of Invariance 400
6.5 Maximal Invariants 402
6.6 Invariance and Noninformative Priors 406
6.6.1 Right and Left Invariant Haar Densities 406
6.6.2 The Best Invariant Rule 409
6.6.3 Confidence and Credible Sets 414
6.7 Invariance and Minimaxity 418
6.8 Admissibility of Invariant Rules 422
6.9 Conclusions 423
Exercises 425

CHAPTER 7
Preposterior and Sequential Analysis 432

7.1 Introduction 432
7.2 Optimal Fixed Sample Size 435
7.3 Sequential Analysis—Notation 441

7.4 Bayesian Sequential Analysis 442
7.4.1 Introduction 442
7.4.2 Notation 445
7.4.3 The Bayes Decision Rule 446
7.4.4 Constant Posterior Bayes Risk 447
7.4.5 The Bayes Truncated Procedure 448
7.4.6 Look Ahead Procedures 455
7.4.7 Inner Truncation 459
7.4.8 Approximating the Bayes Procedure and the Bayes Risk 462
7.4.9 Theoretical Results 467
7.4.10 Other Techniques for Finding a Bayes Procedure 473
7.5 The Sequential Probability Ratio Test 481
7.5.1 The SPRT as a Bayes Procedure 482
7.5.2 Approximating the Power Function and the Expected Sample Size 485
7.5.3 Accuracy of the Wald Approximations 495
7.5.4 Bayes Risk and Admissibility 498
7.5.5 Other Uses of the SPRT 500
7.6 Minimax Sequential Procedures 501
7.7 The Evidential Relevance of the Stopping Rule 502
7.7.1 Introduction 502
7.7.2 The Stopping Rule Principle 502
7.7.3 Practical Implications 504
7.7.4 Criticisms of the Stopping Rule Principle 506
7.7.5 Informative Stopping Rules 510
7.8 Discussion of Sequential Loss Functions 511
Exercises 513

CHAPTER 8
Complete and Essentially Complete Classes 521

8.1 Preliminaries 521
8.2 Complete and Essentially Complete Classes from Earlier Chapters 522
8.2.1 Decision Rules Based on a Sufficient Statistic 522
8.2.2 Nonrandomized Decision Rules 523
8.2.3 Finite Θ 523
8.2.4 The Neyman–Pearson Lemma 523
8.3 One-Sided Testing 525
8.4 Monotone Decision Problems 530
8.4.1 Monotone Multiple Decision Problems 530
8.4.2 Monotone Estimation Problems 534
8.5 Limits of Bayes Rules 537
8.6 Other Complete and Essentially Complete Classes of Tests 538
8.6.1 Two-Sided Testing 538
8.6.2 Higher Dimensional Results 538
8.6.3 Sequential Testing 540
8.7 Complete and Essentially Complete Classes in Estimation 541
8.7.1 Generalized Bayes Estimators 541
8.7.2 Identifying Generalized Bayes Estimators 543

8.8 Continuous Risk Functions 544
8.9 Proving Admissibility and Inadmissibility 546
8.9.1 Stein's Necessary and Sufficient Condition for Admissibility 546
8.9.2 Proving Admissibility 547
8.9.3 Proving Inadmissibility 550
8.9.4 Minimal (or Nearly Minimal) Complete Classes 552
Exercises 554

APPENDIX 1
Common Statistical Densities 559
I Continuous 559
II Discrete 562

APPENDIX 2
Supplement to Chapter 4 563
I Definition and Properties of H_m 563
II Development of (4.121) and (4.122) 564
III Verification of Formula (4.123) 565

APPENDIX 3
Technical Arguments from Chapter 7 568
I Verification of Formula (7.8) 568
II Verification of Formula (7.10) 569

Bibliography 571
Notation and Abbreviations 599
Author Index 603
Subject Index 609