CONTENTS

Preface

Acknowledgments

Glossary of Symbols

	Introduction	1
	Part I BACKGROUND MATERIAL	
1.	Sample Problems	
1.1.	Two-Point Boundary Value Problems	9
	Notes and Remarks	12
	Exercises	13
1.2.	Elliptic Boundary Value Problems	14
	Notes and Remarks	16
1.3.	Integral Equations	18
	Notes and Remarks	20
	Exercises	20
1.4.	Minimization Problems	21
	Notes and Remarks	25
	Exercises	26
1.5.	Two-Dimensional Variational Problems	26
	Notes and Remarks	30
	Exercises	32
	••	

xiii

xvii

xix

viii CONTENTS

2.	Linear Algebra	
2.1.	A Review of Basic Matrix Theory	34
	Notes and Remarks	37
	Exercises	37
2.2.	Norms	38
	Notes and Remarks	44
	Exercises	44
2.3.		45
	Notes and Remarks	50
	Exercises	51
2.4.	0 0	51
	Notes and Remarks	57
	Exercises	57
3.	Analysis	
3.1.	Derivatives and Other Basic Concepts	59
	Notes and Remarks	6:
	Exercises	66
3.2.		68
	Notes and Remarks	73
	Exercises	74
3.3.		74
	Notes and Remarks	81
2.4	Exercises	82
3.4.		82
	Notes and Remarks	88
	Exercises	89
	Part II NONCONSTRUCTIVE EXISTENCE THEOREMS	
4. (Gradient Mappings and Minimization	
4.1.	Minimizers, Critical Points, and Gradient Mappings	9:
	Notes and Remarks	9
	Exercises	9'
4.2.	Uniqueness Theorems	9
	Notes and Remarks	10
	Exercises	102
4.3.		104
	Notes and Remarks	10
	Exercises	10
4.4.	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	110
	Notes and Remarks	11:
	Exercises	11

CONTENTS	ix

٠,	Contractions and the Continuation Property	
5.1.	Contractions	119
	Notes and Remarks	124
	Exercises	125
5.2.	The Inverse and Implicit Function Theorems	125
	Notes and Remarks	131
	Exercises	132
5.3.	The Continuation Property	132
	Notes and Remarks	139
	Exercises	140
5.4.	Monotone Operators and Other Applications	141
	Notes and Remarks	145
	Exercises	145
6.	The Degree of a Mapping	
6.1.	Analytic Definition of the Degree	147
	Notes and Remarks	154
	Exercises	155
6.2.	Properties of the Degree	156
	Notes and Remarks	160
	Exercises	160
6.3.	Basic Existence Theorems	161
	Notes and Remarks	163
	Exercises	164
6.4.	Monotone and Coercive Mappings	165
	Notes and Remarks	167
	Exercises	168
6.5.	APPENDIX. Additional Analytic Results	169
	Part III ITERATIVE METHODS	
7. (General Iterative Methods	
7.1.	Newton's Method and Some of Its Variations	181
	Notes and Remarks	187
	Exercises	188
7.2.	Secant Methods	189
	Notes and Remarks	200
	Exercises	205
7.3.	Modification Methods	206
	Notes and Remarks	212
	Exercises	214
7.4.	Generalized Linear Methods	214
	Notes and Remarks	222
	Exercises	229

X CONTENTS

7.5.	Continuation Methods	230
	Notes and Remarks	234
	Exercises	235
7.6.	General Discussion of Iterative Methods	236
	Notes and Remarks	239
8.	Minimization Methods	
8.1.	Paraboloid Methods	240
	Notes and Remarks	242
	Exercises	243
8.2.		243
	Notes and Remarks	247
	Exercises	248
8.3.		249
	Notes and Remarks	258
0.4	Exercises	259
8.4.	• •	260
	Notes and Remarks Exercises	263
8.5.		266
0.5.	Notes and Remarks	267
	Exercises	269 271
8.6.		2/1
0.0.	Fletcher-Powell Algorithms for Quadratic Functionals	271
8.7.	· · · · · · · · · · · · · · · · · · ·	275
	Part IV LOCAL CONVERGENCE	
9.	Rates of Convergence–General	
9.1.	The Quotient Convergence Factors	281
<i>7.</i> 1.	Notes and Remarks	286
	Exercises	286
9.2.	The Root Convergence Factors	287
	Notes and Remarks	293
	Exercises	294
9.3.	Relations between the R and Q Convergence Factors	295
	Exercises	298
10. (One-Step Stationary Methods	
0.1.	Basic Results	299
	Notes and Remarks	305
	Exercises	308

	Newton's Method and Some of Its Modifications Notes and Remarks Exercises	310 310 318
10.3.	Generalized Linear Iterations Notes and Remarks Exercises	320 331 332
10.4.	Continuation Methods Notes and Remarks	334 340
10.5.	Appendix. Comparison Theorems and Optimal ω for SOR Methods	341
11. 1	Multistep Methods and Additional One-Step Methods	
11.1.	Notes and Remarks	347 352
11.2.	Exercises Consistent Approximations	354 355
	Notes and Remarks Exercises	363 366
11.3.	The General Secant Method	369
	Notes and Remarks Exercises	378 379
12.	Part V SEMILOCAL AND GLOBAL CONVERGENCE Contractions and Nonlinear Majorants	
12. 12.1.	Contractions and Nonlinear Majorants Some Generalizations of the Contraction Theorem Notes and Remarks	383 390 391
	Contractions and Nonlinear Majorants Some Generalizations of the Contraction Theorem	
12.1.	Contractions and Nonlinear Majorants Some Generalizations of the Contraction Theorem Notes and Remarks Exercises Approximate Contractions and Sequences of Contractions Notes and Remarks	390 391 393 398
12.1. 12.2.	Contractions and Nonlinear Majorants Some Generalizations of the Contraction Theorem Notes and Remarks Exercises Approximate Contractions and Sequences of Contractions Notes and Remarks Exercises Iterated Contractions and Nonexpansions Notes and Remarks Exercises Exercises	390 391 393 398 399 400 406
12.1. 12.2. 12.3.	Contractions and Nonlinear Majorants Some Generalizations of the Contraction Theorem Notes and Remarks Exercises Approximate Contractions and Sequences of Contractions Notes and Remarks Exercises Iterated Contractions and Nonexpansions Notes and Remarks Exercises Nonlinear Majorants Notes and Remarks Notes and Remarks	390 391 393 398 399 400 406 407 409 414

CONTENTS

хi

xii CONTENTS

Notes and Remarks 43	13.	Convergence under Partial Ordering	
Exercises	13.1.	Contractions under Partial Ordering	432
13.2. Monotone Convergence 44 Notes and Remarks 44 Exercises 44 13.3. Convexity and Newton's Method 44 Notes and Remarks 45 Exercises 45 13.4. Newton–SOR Interactions 45 Notes and Remarks 46 Exercises 46 13.5. M-Functions and Nonlinear SOR Processes 46 Notes and Remarks 47 Exercises 47 14.1. Introduction and Convergence of Sequences 47 Notes and Remarks 47 Exercises 47 14.2. Steplength Analysis 47 Notes and Remarks 49 Exercises 49 14.3. Gradient and Gradient-Related Methods 49 Notes and Remarks 50 Exercises 50 14.4. Newton-Type Methods 50 Notes and Remarks 50 Exercises 51 14.5. Conjugate-Direction Methods 50 Notes and Remarks 51 Exercises 51 14.6. Univariate Relaxation and Related Pr		Notes and Remarks	439
Notes and Remarks 44		Exercises	440
Exercises	13.2.	<u> </u>	441
13.3. Convexity and Newton's Method 44 Notes and Remarks 45 Exercises 45 13.4. Newton-SOR Interactions 45 Notes and Remarks 46 Exercises 46 13.5. M-Functions and Nonlinear SOR Processes 46 Notes and Remarks 47 Exercises 47 14. Convergence of Minimization Methods 14.1. Introduction and Convergence of Sequences 47 Notes and Remarks 47 Exercises 47 14.2. Steplength Analysis 47 Notes and Remarks 49 Exercises 49 14.3. Gradient and Gradient-Related Methods 49 Notes and Remarks 50 Exercises 50 14.4. Newton-Type Methods 50 Notes and Remarks 50 Exercises 51 14.5. Conjugate-Direction Methods 50 Notes and Remarks 51 Exercises 51 14.6. Univariate Relaxation and Related Processes 51 Notes and Remarks 51 Exercis			446
Notes and Remarks 45	10.0		446
Exercises	13.3.	•	
13.4. Newton–SOR Interactions 45 Notes and Remarks 46 Exercises 46 13.5. M-Functions and Nonlinear SOR Processes 46 Notes and Remarks 47 Exercises 47 14.1. Introduction and Convergence of Sequences 47 Notes and Remarks 47 Exercises 47 14.2. Steplength Analysis 47 Notes and Remarks 49 Exercises 49 14.3. Gradient and Gradient-Related Methods 49 Notes and Remarks 49 Exercises 50 14.4. Newton-Type Methods 50 Notes and Remarks 50 Exercises 50 14.5. Conjugate-Direction Methods 50 Notes and Remarks 51 Exercises 51 14.6. Univariate Relaxation and Related Processes 51 Notes and Remarks 51 Exercises 51 An Annotated List of Basic Reference Books 52 Bibliography 52 Author Index 55			454 454
Notes and Remarks 46	13.4		456
Exercises	13.7.		463
13.5. M-Functions and Nonlinear SOR Processes Notes and Remarks Exercises 14. Convergence of Minimization Methods 14.1. Introduction and Convergence of Sequences Notes and Remarks Exercises 14.2. Steplength Analysis Notes and Remarks Exercises 14.3. Gradient and Gradient-Related Methods Notes and Remarks Exercises 14.4. Newton-Type Methods Notes and Remarks Exercises 14.5. Conjugate-Direction Methods Notes and Remarks Exercises 14.6. Univariate Relaxation and Related Processes Notes and Remarks Exercises 15. Notes and Remarks Exercises 16. Univariate Relaxation and Related Processes Notes and Remarks Exercises 16. Univariate Relaxation and Related Processes Notes and Remarks Exercises 17. Notes and Remarks Exercises 18. Notes and Remarks Exercises 19. Notes and Remarks			464
Notes and Remarks	13.5.		464
14.1. Introduction and Convergence of Sequences Notes and Remarks Exercises 47.2. Steplength Analysis Notes and Remarks Exercises 49.3. Gradient and Gradient-Related Methods Notes and Remarks Exercises 50.4.4. Newton-Type Methods Notes and Remarks Exercises 50.4.5. Conjugate-Direction Methods Notes and Remarks Exercises 50.4.6. Univariate Relaxation and Related Processes Notes and Remarks Exercises 51.6. Univariate Relaxation and Related Processes Notes and Remarks Exercises 51.6. An Annotated List of Basic Reference Books 52. Bibliography 53. Author Index			470
14.1. Introduction and Convergence of Sequences Notes and Remarks Exercises 14.2. Steplength Analysis Notes and Remarks Exercises 14.3. Gradient and Gradient-Related Methods Notes and Remarks Exercises 14.4. Newton-Type Methods Notes and Remarks Exercises 14.5. Conjugate-Direction Methods Notes and Remarks Exercises 14.6. Univariate Relaxation and Related Processes Notes and Remarks Exercises 14.6. Univariate Relaxation and Related Processes Sinuotes and Remarks Exercises 15. An Annotated List of Basic Reference Books 52. Author Index Author Index 53. Action 1.5 Act		Exercises	471
14.1. Introduction and Convergence of Sequences Notes and Remarks Exercises 14.2. Steplength Analysis Notes and Remarks Exercises 14.3. Gradient and Gradient-Related Methods Notes and Remarks Exercises 14.4. Newton-Type Methods Notes and Remarks Exercises 14.5. Conjugate-Direction Methods Notes and Remarks Exercises 14.6. Univariate Relaxation and Related Processes Notes and Remarks Exercises 14.6. Univariate Relaxation and Related Processes Sinuotes and Remarks Exercises 15. An Annotated List of Basic Reference Books 52. Author Index Author Index 53. Action 1.5 Act	14.	Convergence of Minimization Methods	
Notes and Remarks			
Exercises	14.1.		473
14.2. Steplength Analysis Notes and Remarks Exercises 14.3. Gradient and Gradient-Related Methods Notes and Remarks Exercises 14.4. Newton-Type Methods Notes and Remarks Exercises 14.5. Conjugate-Direction Methods Notes and Remarks Exercises 14.6. Univariate Relaxation and Related Processes Notes and Remarks Exercises 15. An Annotated List of Basic Reference Books Author Index Author Index 55.			478
Notes and Remarks Exercises 14.3. Gradient and Gradient-Related Methods Notes and Remarks Exercises 14.4. Newton-Type Methods Notes and Remarks Exercises 14.5. Conjugate-Direction Methods Notes and Remarks Exercises 14.6. Univariate Relaxation and Related Processes Notes and Remarks Exercises 15.1 An Annotated List of Basic Reference Books Author Index 55. Author Index 56.			478
Exercises 14.3. Gradient and Gradient-Related Methods Notes and Remarks Exercises 14.4. Newton-Type Methods Notes and Remarks Exercises 14.5. Conjugate-Direction Methods Notes and Remarks Exercises 14.6. Univariate Relaxation and Related Processes Notes and Remarks Exercises 15.1 An Annotated List of Basic Reference Books 52. Author Index 53. 49. 49. 49. 49. 49. 49. 49. 4	14.2.	- * •	479
14.3. Gradient and Gradient-Related Methods Notes and Remarks Exercises 14.4. Newton-Type Methods Notes and Remarks Exercises 14.5. Conjugate-Direction Methods Notes and Remarks Exercises 14.6. Univariate Relaxation and Related Processes Notes and Remarks Exercises 15. An Annotated List of Basic Reference Books 52. Author Index 55.			493
Notes and Remarks Exercises 14.4. Newton-Type Methods Notes and Remarks Exercises 14.5. Conjugate-Direction Methods Notes and Remarks Exercises 14.6. Univariate Relaxation and Related Processes Notes and Remarks Exercises 15. An Annotated List of Basic Reference Books Author Index 50 49 49 49 49 49 49 49 49 49 4	142		493
Exercises 50 14.4. Newton-Type Methods 50 Notes and Remarks 50 Exercises 50 14.5. Conjugate-Direction Methods 50 Notes and Remarks 51 Exercises 51 14.6. Univariate Relaxation and Related Processes 51 Notes and Remarks 51 Exercises 51 An Annotated List of Basic Reference Books 52 Bibliography 52 Author Index 55	14.5.		494
14.4. Newton-Type Methods Notes and Remarks Exercises 14.5. Conjugate-Direction Methods Notes and Remarks Exercises 14.6. Univariate Relaxation and Related Processes Notes and Remarks Exercises 15.1 An Annotated List of Basic Reference Books 52. Author Index 53. 54. 55. Author Index 56.			
Notes and Remarks Exercises 14.5. Conjugate-Direction Methods Notes and Remarks Exercises 14.6. Univariate Relaxation and Related Processes Notes and Remarks Exercises 15. An Annotated List of Basic Reference Books 52. Bibliography 53. Author Index 54. 55.	11/1		
Exercises 50 14.5. Conjugate-Direction Methods 50 Notes and Remarks 51 Exercises 51 14.6. Univariate Relaxation and Related Processes 51 Notes and Remarks 51 Exercises 51 An Annotated List of Basic Reference Books 52 Bibliography 52 Author Index 55	17.7.	**	
14.5. Conjugate-Direction Methods Notes and Remarks Exercises 14.6. Univariate Relaxation and Related Processes Notes and Remarks Exercises 51 An Annotated List of Basic Reference Books 52 Bibliography 52 Author Index 53 Author Index 55			507
Notes and Remarks Exercises 14.6. Univariate Relaxation and Related Processes Notes and Remarks Exercises 51 An Annotated List of Basic Reference Books 52 Bibliography 52 Author Index 55	14.5		509
Exercises 14.6. Univariate Relaxation and Related Processes Notes and Remarks Exercises 51 An Annotated List of Basic Reference Books 52 Bibliography 52 Author Index 53 54 55 55 Author Index 55	1		512
14.6. Univariate Relaxation and Related Processes Notes and Remarks Exercises 51 An Annotated List of Basic Reference Books 52 Bibliography 52 Author Index 53 Author Index			513
Notes and Remarks 51 Exercises 51 An Annotated List of Basic Reference Books 52 Bibliography 52 Author Index 55	14.6.		513
Exercises 51 An Annotated List of Basic Reference Books 52 Bibliography 52 Author Index 55			518
Bibliography 52 Author Index 55			519
Bibliography 52 Author Index 55		An Annotated Lint of Davis D. C.	
Author Index 55		An Annotated List of Basic Reference Books	521
		Bibliography	523
	Auth	or Index	559
Subject track 56	Subject Index		566