

TABLE OF CONTENTS

1	Introduction	1
2	The Notion of State	5
3	Time-invariant Linear Dynamics	7
	3.1 Continuous time systems	8
	3.2 Inverse systems	11
	3.3 Discrete-time sequences	12
4	Time Series Representation	15
5	Equivalence of ARMA and State Space Models	22
	5.1 AR models	23
	5.2 MA models	24
	5.3 ARMA models	25
	examples	29
6	Decomposition of Data into Cyclical and Growth Components	33
	6.1 Reference paths and variational dynamic models	33
	6.2 Log-linear models as variational models	35
7	Prediction of Time Series	38
	7.1 Prediction space	38
	7.2 Equivalence	44
	7.3 Cholesky decomposition and innovations	45
8	Spectrum and Covariances	48
	8.1 Covariance and spectrum	48
	8.2 Spectral factorization	51
	8.3 Computational aspects	57
	sample covariance matrices	57
	example	58
9	Estimation of System Matrices: Initial Phase	60
	9.1 System matrices	60
	9.2 Approximate model	64

9.3	Rank determination of Hankel matrices: singular value decomposition theorem	67
9.4	Internally balanced model	68
	example	69
	construction	70
	properties of internally balanced models	72
	principal component analysis	74
9.5	Inference about the model order	75
9.6	Choices of basis vectors	77
9.7	State space model	80
	example	81
9.8	ARMA (input-output) model	83
9.9	Canonical correlation	85
10	Innovation Processes	90
10.1	Orthogonal projection	90
10.2	Kalman filters	93
10.3	Innovation model	98
	causal invertibility	101
10.4	Output statistics Kalman filter	102
10.5	Spectral factorization	103
11	Time Series from Intertemporal Optimization	106
11.1	Example: dynamic resource allocation problem	108
11.2	Quadratic regulation problems	116
	discrete-time systems	117
11.3	Parametric analysis of optimal solutions	121
	choice of weighting matrices	122
12	Identification	132
12.1	Closed-loop systems	134
12.2	Identifiability of a closed-loop system	137
13	Time Series from Rational Expectations Models	140
13.1	Moving Average processes	141

13.2	Autoregressive processes	143
13.3	ARMA models	145
13.4	Examples	147
	example 1	147
	example 2	147
	example 3	149
	case of common information pattern	150
	case of differential information set	152
14	Numerical Examples	154
	Mathematical Appendices	178
	A.1 Solutions of difference equations	178
	A.2 Geometry of weakly stationary stochastic sequences	190
	A.3 Principal components	194
	A.4 Fourier transforms	197
	A.5 The z-transform	202
	A.6 Some useful relations for quadratic forms	208
	A.7 Calculation of the inverse, $(zI-A)^{-1}$	211
	A.8 Sensitivity analysis of optimal solutions: scalar-valued case	212
	A.9 Common factor in ARMA models and controllability	215
	A.10 Non-controllability and singular probability distribution	217
	A.11 Spectral decomposition representation	218
	A.12 Singular value decomposition theorem	219
	A.13 Hankel matrices	221
	A.14 Dual relations	228
	A.15 Quadratic regulation problem: continuous time systems	231
	A.16 Maximum principle: discrete-time dynamics	235
	A.17 Policy reaction functions, stabilization policy and modes	237
	A.18 Dynamic policy multipliers	240
	References	242
	Index	248