

# Contents

CHAPTER 1. Populations, Samples, and Random Variables.....	1
1-1. Populations and samples . . .	1
1-2. The random sample . . .	5
1-3. Random variables, probability distributions, and sample spaces . . .	8
1-4. Random digits . . .	14
Bibliography . . .	15
CHAPTER 2. The Normal Distribution.....	16
2-1. The principle of Duhamel and the standard normal distribution . . .	16
2-2. The mean and variance; general normal distribution . . .	21
2-3. Table of the normal distribution . . .	25
2-4. Construction of a normal population . . .	27
2-5. Numerical examples of normal populations (optional) . . .	30
2-6. Random variables from a normal population . . .	34
2-7. Random normal deviates . . .	35
2-8. General continuous populations . . .	36
Bibliography . . .	43
CHAPTER 3. The Normal Sample Space of $n$ Dimensions.....	44
3-1. The geometry of $n$ -dimensional space . . .	45
3-2. Rotations . . .	49
3-3. Integration over $R^n$ . . .	55
3-4. Regular sets . . .	65
3-5. Integration over regular sets . . .	73
3-6. The normal sample space . . .	77
3-7. The sample space for a general population . . .	80
Bibliography . . .	81
CHAPTER 4. Statistics and Their Distributions.....	82
4-1. The definition and role of the statistic . . .	82
4-2. Distribution and density function of a statistic . . .	84
4-3. The chi-square distribution . . .	92

4-4. The fundamental sum-of-squares decomposition and its geometric meaning . . .	96
4-5. The reproductive property of the normal distribution: the distribution of the sum of the coordinates of the sample point . . .	100
4-6. Statistics and their distributions for more general sample spaces (optional) . . .	104
4-7. Numerical illustrations of the distributions of statistics (optional) . . .	108
Bibliography . . .	111
<b>CHAPTER 5. Joint Distributions of Several Statistics: Independence.....</b>	<b>112</b>
5-1. Joint distribution of two statistics: independence . . .	112
5-2. Joint distribution of the sample mean and variance . . .	119
5-3. Distribution of the quotient of two independent statistics . . .	122
5-4. Joint distribution of two or more statistics . . .	129
5-5. Joint distributions of statistics for a general population (optional) . . .	130
Bibliography . . .	131
<b>CHAPTER 6. Spherical Averages, Sufficiency, and Square-Integrable Functions .....</b>	<b>132</b>
6-1. The spherical average of a continuous function . . .	132
6-2. Conditional expectation and sufficiency . . .	137
6-3. Square-integrable functions . . .	141
6-4. Sufficiency and square-integrable functions for more general sample spaces (optional) . . .	148
Bibliography . . .	151
<b>CHAPTER 7. Estimation of Unknown Parameters.....</b>	<b>152</b>
7-1. Statistical background . . .	152
7-2. Formulation of the estimation problem and reduction by sufficiency . . .	156
7-3. Solution of the minimization problem: the Cramér-Rao lower bound . . .	160
7-4. Estimation of $\sigma^2$ when $\mu$ is unknown (optional) . . .	163
7-5. Estimation of parameters for a general population (optional) . . .	167
7-6. Numerical illustrations . . .	173
Bibliography . . .	175
<b>CHAPTER 8. Confidence Intervals.....</b>	<b>176</b>
8-1. Confidence interval for $\mu$ when $\sigma$ is known . . .	176
8-2. Confidence interval for $\mu$ when $\sigma$ is unknown . . .	179
8-3. Stein's two-sample procedure (optional) . . .	183

8-4. Some remarks on confidence intervals for parameters of other densities (optional) . . .	189
8-5. Numerical comparison of confidence intervals based on normal deviates . . .	191
Bibliography . . .	193
<b>CHAPTER 9. Testing Hypotheses</b> .....	<b>194</b>
9-1. Simple hypotheses . . .	194
9-2. The Neyman-Pearson Lemma and the most powerful test . . .	200
9-3. Composite alternative hypothesis: unbiased tests (optional) . . .	206
9-4. Composite hypotheses; invariant tests (optional) . . .	216
9-5. Testing hypotheses for a general population (optional) . . .	225
9-6. Numerical examples . . .	228
Bibliography . . .	229
<b>CHAPTER 10. Sequential Testing (Optional)</b> .....	<b>230</b>
10-1. Fixed sample size and sequential tests . . .	230
10-2. Selection of the "barriers" $A$ and $B$ . . .	236
10-3. Expected sample size in the sequential test . . .	240
10-4. Sequential testing for a general population . . .	245
10-5. A numerical example . . .	249
Bibliography . . .	250
<b>CHAPTER 11. Sampling from Several Normal Populations</b> .....	<b>251</b>
11-1. Sampling from $n$ boxes of balls and $n$ normal populations . . .	251
11-2. A decomposition of a sum of squares and its geometric interpretation . . .	254
11-3. Estimation of a linear function of the means of several normal populations . . .	259
11-4. The noncentral chi-square distribution . . .	267
11-5. Testing homogeneity and invariant tests (optional) . . .	270
11-6. Sampling from several general populations and linear estimation of the means (optional) . . .	279
Bibliography . . .	281
<b>CHAPTER 12. Linear Regression</b> .....	<b>282</b>
12-1. Empirical background of the linear regression model . . .	282
12-2. Least squares and its geometric interpretation . . .	286
12-3. Statistical inference and distribution theory in the linear regression model . . .	290
12-4. Prediction . . .	300
12-5. Linear regression without the assumption of normality . . .	307
12-6. Numerical example of normal linear regression . . .	311
Bibliography . . .	313

CHAPTER 13. One-Way Analysis of Variance.....	314
13-1. Empirical background, replication, sum-of-squares decomposition . . .	314
13-2. Analysis-of-variance test and associated distribution theory . . .	318
13-3. Theoretical justification of analysis-of-variance test: invariance and average power (optional) . . .	326
13-4. Estimation of parameters in the one-way analysis-of-variance model (optional) . . .	340
13-5. A numerical example (optional) . . .	342
13-6. Application to educational testing (optional) . . .	343
Bibliography . . .	345
CHAPTER 14. Correlation and the Bivariate Normal Distribution .....	346
14-1. Definition and empirical background of the bivariate normal distribution . . .	346
14-2. Analytic properties of the bivariate normal distribution . . .	352
14-3. Sampling from a bivariate normal population; the sample correlation coefficient . . .	357
14-4. Testing and hypothesis $\rho = 0$ (optional) . . .	361
14-5. The general bivariate population (optional) . . .	365
14-6. A numerical example of a standard bivariate normal population with $\rho = 1/2$ (optional) . . .	368
Bibliography . . .	369
Appendix .....	371
Index .....	389