

## CONTENTS

Foreword . . . . .	iii
Introduction . . . . .	v
Acknowledgment . . . . .	viii

### Chapter 1. DEFINITIONS AND DISTRIBUTIONS

1.1. Definitions . . . . .	3
1.1.1. Population and Sample . . . . .	3
1.1.2. Distribution . . . . .	4
1.1.3. Measures of Central Location . . . . .	10
a. Mean . . . . .	10
b. Median . . . . .	11
c. Mode . . . . .	12
d. Midrange . . . . .	12
1.1.4. Measures of Dispersion . . . . .	12
a. Standard Deviation . . . . .	12
b. Range . . . . .	13
c. Mean Deviation . . . . .	13
d. Various Percentile Estimates . . . . .	13
e. Probable Error . . . . .	13
1.1.5. Coding . . . . .	14
1.1.6. Sampling Distribution . . . . .	14
1.1.7. Test of a Statistical Hypothesis . . . . .	15
1.1.8. Level of Significance $\alpha$ and Types of Error . . . . .	16
1.1.9. Operating Characteristic Curve . . . . .	17
1.1.10. Confidence Intervals . . . . .	18
1.1.11. Tolerance Intervals . . . . .	18
1.1.12. Degrees of Freedom . . . . .	19
1.2. Particular Distributions . . . . .	19
1.2.1. Normal Distribution . . . . .	19
1.2.2. Binomial Distribution . . . . .	30
1.2.3. Poisson Distribution . . . . .	32
Bibliography . . . . .	35

**Chapter 2. TESTS AND CONFIDENCE INTERVALS  
FOR MEANS**

<b>2.0. Introduction . . . . .</b>	<b>37</b>
<b>2.1. Test for Mean <math>\mu</math> When Standard Deviation <math>\sigma</math> Is Known . . . . .</b>	<b>38</b>
2.1.1. Assumptions . . . . .	38
2.1.2. Normal Test . . . . .	38
a. Determination of Sample Size . . . . .	40
2.1.3. Confidence Intervals . . . . .	43
a. Determination of Sample Size . . . . .	43
<b>2.2. Test for <math>\mu</math> When <math>\sigma</math> Is Unknown . . . . .</b>	<b>44</b>
2.2.1. The $\tau_1$ Test . . . . .	44
2.2.2. The $t$ Test . . . . .	45
a. Determination of Sample Size . . . . .	46
2.2.3. Confidence Intervals . . . . .	47
a. Determination of Sample Size . . . . .	48
<b>2.3. Test for Proportion <math>p</math> . . . . .</b>	<b>48</b>
2.3.1. Assumptions . . . . .	49
2.3.2. Test . . . . .	49
a. Test for a Sample of Size $n \leq 150$ . . . . .	49
b. Test for a Sample of Size $n > 5/p_0$ . . . . .	50
c. Determination of Sample Size . . . . .	51
2.3.3. Confidence Intervals . . . . .	51
a. Graphical and Tabular Solutions . . . . .	51
b. Numerical Solution . . . . .	52
<b>2.4. Test for <math>\mu_1 - \mu_2</math> When <math>\sigma_1</math> and <math>\sigma_2</math> Are Known . . . . .</b>	<b>52</b>
2.4.1. Test Conditions and Assumptions . . . . .	52
2.4.2. Normal Test . . . . .	53
a. Test for Paired Observations . . . . .	53
b. Test for Two Independent Samples . . . . .	53
c. Determination of Size of Independent Samples . . . . .	54
2.4.3. Confidence Intervals . . . . .	55
a. Determination of Size of Independent Samples . . . . .	55
<b>2.5. Test for <math>\mu_1 - \mu_2</math> When <math>\sigma_1</math> and <math>\sigma_2</math> Are Unknown . . . . .</b>	<b>55</b>
2.5.1. Assumptions and Design of Experiment . . . . .	55
2.5.2. Test for Paired Observations . . . . .	56
a. Sign Test . . . . .	56
b. The $t$ Test on Differences . . . . .	57
2.5.3. Test for Two Independent Samples When $\sigma_1 = \sigma_2$ . . . . .	57
a. The $\tau_d$ Test . . . . .	57
b. The $t$ Test . . . . .	57

## CONTENTS

2.5.4.	Test of $\mu_1 - \mu_2 = 0$ for Two Independent Samples When $\sigma_1 \neq \sigma_2$	60
2.6.	Test for a Difference Between Proportions, $p_1 - p_2$	62
2.7.	Test for Differences Among Several Means	62
2.8.	Detection of a Trend in a Set of Means, Standard Deviations Unknown but Assumed Equal	62
2.8.1.	Assumptions	62
2.8.2.	Method	63
Bibliography		64
<b>Chapter 3. TESTS AND CONFIDENCE INTERVALS FOR STANDARD DEVIATIONS</b>		
3.0.	Introduction	67
3.1.	Computation of $s$	68
3.2.	$\chi^2$ Test for $\sigma$ in a Normal Distribution	70
3.2.1.	Assumptions	70
3.2.2.	Test	70
	a. Determination of Sample Size	72
3.2.3.	Confidence Intervals	73
	a. Determination of Sample Size	73
3.3.	$F$ Test for $\sigma_1/\sigma_2$ for Normal Distributions	74
3.3.1.	Assumptions	74
3.3.2.	Test	74
	a. Determination of Sample Size	77
3.3.3.	Confidence Intervals	77
	a. Determination of Sample Size	78
3.4.	$M$ Test for Homogeneity of Variances	78
3.4.1.	Assumptions	78
3.4.2.	Test	78
Bibliography		81
<b>Chapter 4. TESTS OF DISTRIBUTIONS AS A WHOLE AND ALLIED PROBLEMS</b>		
4.0.	Introduction	83
4.1.	Run Test for Randomness	83
4.2.	The $\chi^2$ Test for Goodness of Fit	85
4.3.	Testing the Fit of a Normal Distribution	87
4.3.1.	Transformations To Obtain Normality	88
4.4.	Confidence Band for Cumulative Distribution	90
4.5.	Sensitivity Testing	91
4.5.1.	Up-and-Down Method	93

## CONTENTS

4.6. Test for Independence in Contingency Tables . . . . .	97
4.7. Consistency of Several Samples . . . . .	100
4.7.1. The $\chi^2$ Test . . . . .	100
4.7.2. Run Test for Comparing Two Samples . . . . .	101
4.8. Gross Errors . . . . .	102
4.9. Tolerance Intervals . . . . .	104
Bibliography . . . . .	106

**Chapter 5. PLANNING OF EXPERIMENTS AND ANALYSIS  
OF VARIANCE**

5.0. Introduction . . . . .	109
5.1. Common Designs for Experiments . . . . .	111
5.1.1. Complete Factorial Design for Treatments . . . . .	114
5.1.2. Complete Randomization for Background Conditions .	115
5.1.3. Randomized-Block Design for Background Conditions .	116
5.1.4. Latin-Square Design for Background Conditions . .	116
5.1.5. Other Designs . . . . .	117
5.2. Types of Analysis of Variance . . . . .	118
5.2.1. Type I Problems . . . . .	118
5.2.2. Type II Problems . . . . .	118
5.3. Assumptions of Analysis of Variance . . . . .	119
5.4. One-Factor Analysis . . . . .	120
5.4.1. Assumptions . . . . .	120
5.4.2. Analysis . . . . .	122
5.5. Two-Factor Analysis . . . . .	127
5.5.1. Two-Factor Analysis Without Replication . . . . .	127
5.5.2. Two-Factor Analysis With Replication . . . . .	133
5.6. Multiple-Factor Analysis . . . . .	139
Bibliography . . . . .	144

**Chapter 6. FITTING A FUNCTION OF ONE OR  
MORE VARIABLES**

6.0. Introduction . . . . .	147
6.1. Linear Regression (Two Variables) . . . . .	152
6.1.1. Regression Line . . . . .	152
6.1.2. Variation About the Regression Line . . . . .	156
6.1.3. Fraction of $y$ Variance Accounted for by Regression—Correlation Coefficient $r$ . . . . .	157

6.1.4.	Reliability of Regression Measures . . . . .	158
a.	Significance of Sample Correlation Coefficient $r$ . . . . .	159
b.	Confidence Interval for Population Correlation Coefficient $\rho$ . . . . .	159
c.	Significance Test and Confidence Interval for the Slope $b$ of the Regression Line . . . . .	160
d.	Significance of the Difference Between the Regression Coefficients $b_1$ and $b_2$ of Two Separate Equations . . . . .	161
e.	Confidence Interval for an Ordinate to the True Regression Line . . . . .	162
f.	Prediction Interval for Individual Value of $y$ . . . . .	163
6.1.5.	Summary of Computations for Two-Variable Linear Regression . . . . .	164
6.1.6.	Test for Linearity of Regression . . . . .	165
6.2.	Multiple Linear Regression . . . . .	168
6.2.1.	Multiple Linear Equation . . . . .	169
6.2.2.	Determination of Partial Regression Coefficients . . . . .	170
6.2.3.	Variation About the Regression Plane . . . . .	174
6.2.4.	Relative Variation (Type II Problem) . . . . .	175
a.	Multiple Correlation Coefficient . . . . .	175
b.	Partial Correlation Coefficients . . . . .	176
6.2.5.	Reliability of Multiple Regression Measures . . . . .	177
a.	Significance of Regression as a Whole . . . . .	177
b.	Significance of Multiple Correlation Coefficient . . . . .	178
c.	Significance of Partial Correlation Coefficients . . . . .	178
d.	Confidence Intervals for Partial Correlation Coefficients . . . . .	179
e.	Significance Tests and Confidence Intervals for Partial Regression Coefficients . . . . .	179
f.	Confidence Interval for Ordinate to the True Regression Plane . . . . .	181
g.	Prediction Interval for Individual Value of $y$ . . . . .	182
6.3.	Non-Linear Regression . . . . .	183
6.3.1.	Polynomial Equations . . . . .	185
6.3.2.	Orthogonal Polynomials, When $x$ is Equally Spaced . . . . .	186
6.4.	Planning the Experiment for Fitting a Function . . . . .	191
Bibliography . . . . .		193
<b>Chapter 7. QUALITY-CONTROL CHARTS</b>		
7.0.	Introduction . . . . .	195
7.1.	Charts Using Attributes . . . . .	199

## CONTENTS

7.2. Charts Using Variables . . . . .	202
Bibliography . . . . .	205
 Chapter 8. ACCEPTANCE SAMPLING	
8.0. Introduction . . . . .	209
8.1. Attributes or Variables . . . . .	210
8.2. Choice of Plan . . . . .	211
8.2.1. Acceptable-Quality Level (AQL) . . . . .	211
8.2.2. Single, Double, or Multiple Sampling . . . . .	212
8.2.3. Inspection Level . . . . .	213
8.2.4. Sample Size and Severity of Inspection . . . . .	214
8.2.5. Plans for Normal Inspection . . . . .	215
a. Attributes . . . . .	215
b. Variables . . . . .	216
8.3. Operating-Characteristic (OC) Curve and Its Use in Designing Plans . . . . .	217
8.4. Sequential Plans . . . . .	220
8.4.1. Sequential Plan for Proportion Defective $p$ . . . . .	221
8.4.2. Sequential Plan for Standard Deviation $\sigma$ . . . . .	224
Bibliography . . . . .	225
 Appendix Tables:	
1. Cumulative Normal Distribution . . . . .	229
2. Percentiles of the Normal Distribution . . . . .	230
3. Upper Percentage Points of the $t$ Distribution . . . . .	231
4. Upper Percentage Points of the $\chi^2$ Distribution . . . . .	232
5. Critical Values for the $F$ Distribution . . . . .	234
6. Critical Values for the $M$ Distribution at the 5% Significance Level . . . . .	240
7. Critical Absolute Values of Correlation Coefficient $r$ . . . . .	241
8. Factors for Determining Confidence Limits for Population Standard Deviation $\sigma$ . . . . .	242
9. Critical Values for the Sign Test . . . . .	243
10. Critical Values for Runs . . . . .	244
11. Half-Widths $d_a$ for Construction of Confidence Bands for Cumulative Distributions . . . . .	248
12. Factor $k$ for Estimating Normal Standard Deviation From the Range $w$ as $kw$ . . . . .	248

13. Critical Values for Tests Using the Range $w$	249
(a) Critical Values for $\tau_1 = \frac{\bar{x} - a}{w}$	249
(b) Critical Values for $\tau_d = \frac{\bar{x}_1 - \bar{x}_2}{w_1 + w_2}$	249
14. Critical Values for $\delta^2/f^2$	250
15. Orthogonal Polynomials	251
16. Critical Values for Test Ratios for Gross Errors	252
17. Tolerance Factors $K$ for Population Proportion $P$ of Normal Distributions	253
18. Factors for Computing Control-Chart Lines	254
19. Constants for a Sampling Plan With OC Curve Through $(p_1, 0.95)$ and $(p_2, 0.05)$	255
(a) Single Sampling Plan	255
(b) Double Sampling Plan	255
20. Table of $a = \log_{10} \frac{1 - \beta}{\alpha}$ and $b = \log_{10} \frac{1 - \alpha}{\beta}$	256
21. Confidence Limits for a Proportion	257
(a) Two-Sided Limits	257
(b) One-Sided Limits	262
 Appendix Charts:	
I. Rectangular Normal Probability Chart	269
II. 90% Confidence Belts for Proportions	270
III. 95% Confidence Belts for Proportions	271
IV. 99% Confidence Belts for Proportions	272
V. OC Curves for Testing the Hypothesis $\mu = a$ by the Equal-Tails Normal Test	273
VI. OC Curves for Testing the Hypothesis $\mu = a$ by the Equal-Tails $t$ Test	274
VII. OC Curves for Testing the Hypothesis $\sigma = \sigma_0$ Against $\sigma = \sigma_1 > \sigma_0$ by the $\chi^2$ Test	275
VIII. OC Curves for Testing the Hypothesis $\sigma_1 = \sigma_2$ Against $\sigma_1 > \sigma_2$ by the $F$ Test	276
IX. Number of Degrees of Freedom Required To Estimate the Standard Deviation Within $p\%$ of Its True Value, With Prescribed Confidence	277
X. Number of Degrees of Freedom in Each Sample Required To Estimate $\sigma_1/\sigma_2$ Within $p\%$ of Its True Value, With Prescribed Confidence	278
XI. 95% Confidence Belts for Correlation Coefficient	279
Index	281