

Contents

<i>Preface</i>	<i>page</i> ix
1 Elementary operator theory	1
1.1 Banach spaces	1
1.2 Bounded linear operators	12
1.3 Topologies on vector spaces	19
1.4 Differentiation of vector-valued functions	23
1.5 The holomorphic functional calculus	27
2 Function spaces	35
2.1 L^p spaces	35
2.2 Operators acting on L^p spaces	45
2.3 Approximation and regularization	54
2.4 Absolutely convergent Fourier series	60
3 Fourier transforms and bases	67
3.1 The Fourier transform	67
3.2 Sobolev spaces	77
3.3 Bases of Banach spaces	80
3.4 Unconditional bases	90
4 Intermediate operator theory	99
4.1 The spectral radius	99
4.2 Compact linear operators	102
4.3 Fredholm operators	116
4.4 Finding the essential spectrum	124

5	Operators on Hilbert space	135
5.1	Bounded operators	135
5.2	Polar decompositions	137
5.3	Orthogonal projections	140
5.4	The spectral theorem	143
5.5	Hilbert-Schmidt operators	151
5.6	Trace class operators	153
5.7	The compactness of $f(Q)g(P)$	160
6	One-parameter semigroups	163
6.1	Basic properties of semigroups	163
6.2	Other continuity conditions	177
6.3	Some standard examples	182
7	Special classes of semigroup	190
7.1	Norm continuity	190
7.2	Trace class semigroups	194
7.3	Semigroups on dual spaces	197
7.4	Differentiable and analytic vectors	201
7.5	Subordinated semigroups	205
8	Resolvents and generators	210
8.1	Elementary properties of resolvents	210
8.2	Resolvents and semigroups	218
8.3	Classification of generators	227
8.4	Bounded holomorphic semigroups	237
9	Quantitative bounds on operators	245
9.1	Pseudospectra	245
9.2	Generalized spectra and pseudospectra	251
9.3	The numerical range	264
9.4	Higher order hulls and ranges	276
9.5	Von Neumann's theorem	285
9.6	Peripheral point spectrum	287
10	Quantitative bounds on semigroups	296
10.1	Long time growth bounds	296
10.2	Short time growth bounds	300
10.3	Contractions and dilations	307
10.4	The Cayley transform	310

10.5	One-parameter groups	315
10.6	Resolvent bounds in Hilbert space	321
11	Perturbation theory	325
11.1	Perturbations of unbounded operators	325
11.2	Relatively compact perturbations	330
11.3	Constant coefficient differential operators on the half-line	335
11.4	Perturbations: semigroup based methods	339
11.5	Perturbations: resolvent based methods	350
12	Markov chains and graphs	355
12.1	Definition of Markov operators	355
12.2	Irreducibility and spectrum	359
12.3	Continuous time Markov chains	362
12.4	Reversible Markov semigroups	366
12.5	Recurrence and transience	369
12.6	Spectral theory of graphs	374
13	Positive semigroups	380
13.1	Aspects of positivity	380
13.2	Invariant subsets	386
13.3	Irreducibility	390
13.4	Renormalization	393
13.5	Ergodic theory	395
13.6	Positive semigroups on $C(X)$	399
14	NSA Schrödinger operators	408
14.1	Introduction	408
14.2	Bounds on the numerical range	409
14.3	Bounds in one space dimension	412
14.4	The essential spectrum of Schrödinger operators	420
14.5	The NSA harmonic oscillator	424
14.6	Semi-classical analysis	427
	<i>References</i>	436
	<i>Index</i>	446