# Contents


Preface xiii

PART I
BASIC TOOLS OF MULTIVARIATE DATA ANALYSIS 1

Chapter 1 The Data Matrix 3
- Introduction 3
- 1.1 Writing the Data Matrix 3
- 1.2 Predictors and Criteria 5
- 1.3 Measured and Categorical Observations 11
- 1.4 Selecting vs Sampling Observations 17
- 1.5 Additional Data Sets Discussed in This Book 22
- 1.6 Partitioned Matrices and Computer Analyses 23
- 1.7 Summary 25
- Exercises 25
- Additional Reading 26

Chapter 2 Working with Matrices 27
- Introduction 27
- 2.1 Types of Matrices 27
  - 2.1.1 Matrix Dimensions 28
  - 2.1.2 Matrix Transposition 29
- 2.2 Special Matrices and Vectors 32
- 2.3 Addition and Subtraction of Matrices 37
- 2.4 Matrix Products 40
  - 2.4.1 Matrix Products and Vectors 43
  - 2.4.2 Partitioned Products 46
  - 2.4.3 Scaling 49

2.5 Special Matrix Products 50
2.5.1 Product Moments 50
2.5.2 Unit Vector Products 53
2.5.3 Elementary Vector Products 54
2.5.4 Matrix Centering 55
2.5.5 Variance–Covariance Matrices 58
2.5.6 Diagonal Products 60
2.5.7 Permutation of Rows or Columns 62
2.6 Matrix Operations and the Computer 63
2.7 Summary 64
Exercises 65
Additional Reading 67

Chapter 3 The Information in a Matrix 68
Introduction 68
3.1 Metrics and Transformations 69
3.2 Dilemma of Standardization 74
3.3 Amount of Information in a Matrix 75
3.3.1 Rank 75
3.3.2 Basic and Nonbasic Matrices 77
3.3.3 Orthogonal and Orthonormal Matrices 80
3.3.4 Two Rank Inequalities 82
3.4 Finding the Rank of a Matrix 86
3.4.1 Triangular Factoring 87
3.4.2 Nonbasic Example 93
3.5 Regular Inverse 96
3.5.1 Finding Inverses Using Triangular Factoring 100
3.5.2 Inversion of Symmetrically Partitioned Matrices 104
3.6 Summary 106
Exercises 108
Additional Reading 109

Chapter 4 The Basic Structure of a Matrix 110
Introduction 110
4.1 Some Factorings of a Data Matrix 110
4.2 Definition of the Basic Structure of a Matrix 114
4.3 Basic Structure as a Transformation of the Data Matrix 117
4.4 Uniqueness of the Basic Structure 118
4.5 Basic Structures of Special Matrices 119
4.5.1 Basic Structures of Product Moments 119
4.5.2 Basic Structure and the Regular Inverse 120
4.5.3 Basic Structure and a Generalized Inverse 121
4.5.4 Basic Structure of Types of Matrices 123
4.5.5 Powers of Product Moments 127

4.6 Finding the Basic Structure of a Matrix 127
4.6.1 Matrix Powering and Basic Structure 128
4.6.2 Example 130
4.7 Some Notes on Computation 134
4.8 Summary 134
Exercises 135
Additional Reading 136

Chapter 5 Transforming Data Matrices 137
Introduction 137
5.1 Multivariate Analysis: Directed Data Transformations 138
5.2 Linear Transformations of Attributes 139
5.3 Rank of a Set of Linear Transformations 140
5.4 Basic Structure of Transforming Matrices 142
5.5 Rank Reduction Transformation 144
5.6 Quadratic and Bilinear Transformations 148
5.7 Spherizing Transformations 153
5.8 Summary 157
Exercises 157
Additional Reading 158

Chapter 6 Optimizing Transformations 159
Introduction 159
6.1 Multivariate Analyses and Optimizing Transformations 160
6.2 Elementary Derivatives, Maxima, and Minima 160
6.3 Rules of Differentiation 166
6.3.1 Algebraic Functions 166
6.3.2 Exponential and Logarithmic Functions 167
6.3.3 Functions of Functions 168
6.4 Partial Differentiation 169
6.5 Side Conditions in Optimizing Transformations 171
6.6 Matrix Differentiation and Optimization 174
6.6.1 Differentiation of a Matrix with Respect to a Scalar 174
6.6.2 Differentiation of a Scalar with Respect to a Matrix 175
6.6.3 Differentiation of a Matrix with Respect to a Matrix 175
6.6.4 Symbolic Derivatives 176
6.7 Summary 180
Exercises 181
Additional Reading 181

PART II
BASIC STRUCTURE AND THE UNIVARIATE CRITERION 183

Chapter 7 Least Squares: Data Fitting 185
Introduction 185
7.1 Least Squares Fitting Procedure 186
7.1.1 Least Squares with a Nonbasic Predictor Matrix 190

7.2 Constraining the Least Squares Weights 193
7.2.1 Constraint Matrix 195
7.2.2 Least Squares Solution with Constraints 196
7.3 Data-Bound Multiple and Partial Correlations 200
7.3.1 Squared Multiple Correlation 202
7.3.2 Partial Correlation 206
7.4 Summary 212
Exercises 213
Additional Reading 214

Chapter 8 Linear Regression: Some Theory 215
Introduction 215
8.1 Linear Regression Problem 216
8.2 Maximum Likelihood Estimation of Parameters 219
8.3 Sampling Distribution of Maximum Likelihood Estimators 225
8.4 Hypothesis Testing 231
8.4.1 Testing Hypotheses About $\sigma^2$ 231
8.4.2 Likelihood Ratio Test of Hypotheses About $\boldsymbol{\beta}$ 235
8.4.3 Testing Constraints on $\boldsymbol{\beta}$ 240
8.4.4 Multiple Hypotheses on $\boldsymbol{\beta}$ 244
8.4.5 Interval Estimates of $\boldsymbol{\beta}$ 251
8.5 Summary 253
Exercises 254
Additional Reading 254

Chapter 9 Linear Regression: Some Applications 255
Introduction 255
9.1 Fixed Effects Analysis of Variance Designs 255
9.1.1 Design Template 257
9.1.2 ANOVA Sums of Squares 264
9.1.3 Hypothesis Testing in Randomized Groups ANOVA 266
9.1.4 Factorial Design 268
9.2 Concomitant Variable and Covariance Analysis 277
9.3 Multiple Linear Regression Applications 280
9.3.1 Curve Fitting 281
9.3.2 Polynomial Regression 283
9.3.3 Testing for Linearity 286
9.3.4 Moderator Variables 288
9.4 Summary 290
Exercises 291
Additional Reading 293

PART III
BASIC STRUCTURE AND THE MULTIVARIATE CRITERION 295

Chapter 10 Basic Structure and Multivariate Normal Samples 297
Introduction 297
10.1 Jointly Distributed Observations 297

10.2 Multivariate Normal Distribution 298
10.3 Generalized Variance 302
10.4 Population Correlation and Bivariate Normal Distributions 304
10.4.1 Marginal and Conditional Distributions 306
10.5 Marginal and Conditional Distributions Based on the Multivariate Normal 313
10.5.1 Marginal Distributions 313
10.5.2 Conditional Distributions 313
10.6 Multivariate Partial and Multiple Correlation 319
10.6.1 Population Partial Correlation Matrix 319
10.6.2 Population Multiple Correlation 320
10.7 Parameter Estimation for the Multivariate Normal Distribution 321
10.7.1 Maximum Likelihood Estimator of $\boldsymbol{\mu}$ 322
10.7.2 Maximum Likelihood Estimator of $\boldsymbol{\Sigma}$ 322
10.7.3 Distribution of the Maximum Likelihood Estimator for $\boldsymbol{\mu}$ 324
10.7.4 Expected Value of $\hat{\boldsymbol{\Sigma}}$ 325
10.8 A Multivariate Generalization of Chi-Squared 326
10.8.1 Wishart Matrices 327
10.8.2 Distribution of Products of Basic Diagonals 330
10.9 Summary 336
Exercises 337
Additional Reading 338

Chapter 11 The Multivariate General Linear Model: Some Principles 339
Introduction 339
11.1 Sampling from Several Multivariate Normal Populations 339
11.1.1 Review of the Univariate General Linear Model 340
11.1.2 Framework of the Multivariate General Linear Model 341
11.1.3 Maximum Likelihood Estimation of $\boldsymbol{\beta}$ 342
11.1.4 Maximum Likelihood Estimation of $\boldsymbol{\Sigma}$ 343
11.1.5 Some Sampling Distribution Results for the Estimators of $\boldsymbol{\beta}$ and $\boldsymbol{\Sigma}$ 343
11.1.6 Wishart Distributions in the MGLM 345
11.2 Effects of Linear Constraints 346
11.2.1 Two-Group $T^2$ 346
11.2.2 Randomized Groups Multivariate Analysis of Variance 347
11.2.3 Multivariate Linear Regression 348
11.2.4 Known Population Mean Vectors 349
11.2.5 Constraining $\boldsymbol{\beta}$ by Column 350
11.3 Hypothesis Testing in the Multivariate General Linear Model 353
11.3.1 Maximum Likelihood Estimation for $\mathbf{C}'\boldsymbol{\beta}\mathbf{A} = \boldsymbol{\Gamma}$ 353
11.3.2 Sums of Squares of Errors for the Effective Criterion 356
11.3.3 Sampling Distributions of Multivariate Error Sums of Squares 356
11.3.4 Likelihood Ratio Basis for Hypothesis Sampling 358

11.3.5 Wilks's Lambda and the $U$ Distribution 360
11.3.6 Union–Intersection Basis for Hypothesis Testing 361
11.3.7 Roy's Greatest Basic Diagonal Test 363
11.4 Summary 366
Exercises 367
Additional Reading 367

Chapter 12 The Multivariate General Linear Model: Some Applications 368
Introduction 368
12.1 Examples of MGLM Problems 368
12.1.1 Two-Group $T^2$ Example 368
12.1.2 Randomized Groups Multivariate Analysis of Variance 370
12.1.3 Multivariate Linear Regression Example 373
12.1.4 Combining Criterion Variables 375
12.2 Linear Discriminant Function 377
12.2.1 Linear Discriminant Problem 377
12.2.2 Sample Solution to the Linear Discriminant Function 379
12.2.3 Significance of the Linear Discriminant Function 380
12.2.4 Additional Linear Discriminant Functions Beyond the First 381
12.2.5 Classifying New Observations 385
12.2.6 LDF Example: Vocational Interest and College Major 386
12.3 Homogeneity of Variance–Covariance Matrices 389
12.4 Summary 392
Exercises 392
Additional Reading 393

PART IV
MULTIVARIATE APPROACHES AND OTHER APPLICATIONS 395

Chapter 13 An Analysis Model for Categorical Data 397
Introduction 397
13.1 Three Sampling Models for Obtaining Frequencies 398
13.1.1 Sampling with No Marginal Restrictions 398
13.1.2 Sampling to Obtain a Fixed Total Number of Observations 399
13.1.3 Sampling to Obtain Fixed Marginal Totals 400
13.2 General Log Linear Model for Cell Frequencies 401
13.2.1 Some Likelihood Ratios 401
13.2.2 Kullback's Discrimination Information Statistic 403
13.2.3 A Class of Models 404
13.2.4 Internal and External Constraints 404
13.2.5 Internal Constraints Problem and Minimum Discrimination Information 405
13.2.6 Iterative Proportional Fitting of Marginals 410
13.2.7 Log Linear Computations 412
13.2.8 Partitioning the Discrimination Information Statistic 413
13.3 Hierarchical Internal Constraints Problem 416
13.3.1 Factorial Design 417
13.3.2 Log Linear Model for Cross Classified Frequencies 421
13.3.3 Job Satisfaction Example 424

13.4 Log Linear External Constraints Problem 426
13.4.1 MDI Solution for External Constraints 428
13.4.2 Iterative Solution to the External Constraints Problem 429
13.4.3 Application of the Newton–Raphson Iteration 432
13.4.4 Goodness of Fit for the External Constraints Problem 434
13.5 Summary 434
Exercises 435
Additional Reading 436

Chapter 14 Basic Structure and the Analysis of Interdependence 437
Introduction 437
14.1 Basic Structure and Principal Components Analysis 437
14.2 Solving for the Principal Components 440
14.3 Application of Principal Components 440
14.4 Further Interpretation of Principal Components 443
14.5 Principal Components as a Method of Orthogonalization 446
14.6 Computer Solutions and Principal Components Analysis 446
14.7 Principal Components and Factor Analysis 446
14.8 Principal Components and Factor Analysis: A Comparison 448
14.9 Regression on Principal Components 448
14.10 Summary 450
Exercises 451
Additional Reading 452

Chapter 15 Latent Variable Structural Equation Models 453
Introduction 453
15.1 JKW Latent Variable Structural Equation Model 454
15.1.1 Measurement Models 455
15.1.2 Structural Equation Models 456
15.1.3 Variance–Covariance Matrix of the Observed Attributes 457
15.1.4 Identifiability in Latent Variable Models 458
15.1.5 Estimation in Latent Variable Models 459
15.1.6 Interpreting the Results of Estimation 460
15.2 Some Applications of the JKW Model 461
15.3 Special Cases of the JKW Model 466
15.3.1 Factor Analysis 466
15.3.2 Linear Regression on Observed Attributes 467
15.3.3 Population Correlation Models 467
15.3.4 Finding Principal Components with LISREL 467
15.4 Summary 467
Exercises 468
Additional Reading 469

Appendix 471

Index 517