

# Contents

Preface	vii
CHAPTER 1	
Introduction to the Power-Divergence Statistic	1
1.1 A Unified Approach to Model Testing	1
1.2 The Power-Divergence Statistic	2
1.3 Outline of the Chapters	3
CHAPTER 2	
Defining and Testing Models: Concepts and Examples	5
2.1 Modeling Discrete Multivariate Data	5
2.2 Testing the Fit of a Model	9
2.3 An Example: Time Passage and Memory Recall	12
2.4 Applying the Power-Divergence Statistic	14
2.5 Power-Divergence Measures in Visual Perception	17
CHAPTER 3	
Modeling Cross-Classified Categorical Data	19
3.1 Association Models and Contingency Tables	19
3.2 Two-Dimensional Tables: Independence and Homogeneity	20
3.3 Loglinear Models for Two and Three Dimensions	25
3.4 Parameter Estimation Methods: Minimum Distance Estimation	28
3.5 Model Generation: A Characterization of the Loglinear, Linear, and Other Models through Minimum Distance Estimation	34
3.6 Model Selection and Testing Strategy for Loglinear Models	40

CHAPTER 4	
Testing the Models: Large-Sample Results	44
4.1 Significance Levels under the Classical (Fixed-Cells) Assumptions	45
4.2 Efficiency under the Classical (Fixed-Cells) Assumptions	53
4.3 Significance Levels and Efficiency under Sparseness Assumptions	57
4.4 A Summary Comparison of the Power-Divergence Family Members	62
4.5 Which Test Statistic?	63
CHAPTER 5	
Improving the Accuracy of Tests with Small Sample Size	64
5.1 Improved Accuracy through More Accurate Moments	64
5.2 A Second-Order Correction Term Applied Directly to the Asymptotic Distribution	68
5.3 Four Approximations to the Exact Significance Level: How Do They Compare?	69
5.4 Exact Power Comparisons	76
5.5 Which Test Statistic?	79
CHAPTER 6	
Comparing the Sensitivity of the Test Statistics	81
6.1 Relative Deviations between Observed and Expected Cell Frequencies	81
6.2 Minimum Magnitude of the Power-Divergence Test Statistic	83
6.3 Further Insights into the Accuracy of Large-Sample Approximations	86
6.4 Three Illustrations	88
6.5 Transforming for Closer Asymptotic Approximations in Contingency Tables with Some Small Expected Cell Frequencies	92
6.6 A Geometric Interpretation of the Power-Divergence Statistic	94
6.7 Which Test Statistic?	96
CHAPTER 7	
Links with Other Test Statistics and Measures of Divergence	98
7.1 Test Statistics Based on Quantiles and Spacings	99
7.2 A Continuous Analogue to the Discrete Test Statistic	103
7.3 Comparisons of Discrete and Continuous Test Statistics	105
7.4 Diversity and Divergence Measures from Information Theory	106
CHAPTER 8	
Future Directions	114
8.1 Hypothesis Testing and Parameter Estimation under Sparseness Assumptions	114
8.2 The Parameter $\lambda$ as a Transformation	118
8.3 A Generalization of Akaike's Information Criterion	124
8.4 The Power-Divergence Statistic as a Measure of Loss and a Criterion for General Parameter Estimation	128
8.5 Generalizing the Multinomial Distribution	132

Historical Perspective: Pearson's $X^2$ and the Loglikelihood Ratio Statistic $G^2$	133
1. Small-Sample Comparisons of $X^2$ and $G^2$ under the Classical (Fixed-Cells) Assumptions	134
2. Comparing $X^2$ and $G^2$ under Sparseness Assumptions	140
3. Efficiency Comparisons	144
4. Modified Assumptions and Their Impact	150
 Appendix: Proofs of Important Results	 154
A1. Some Results on Rao Second-Order Efficiency and Hodges-Lehmann Deficiency (Section 3.4)	154
A2. Characterization of the Generalized Minimum Power-Divergence Estimate (Section 3.5)	159
A3. Characterization of the Lancaster-Additive Model (Section 3.5)	160
A4. Proof of Results (i), (ii), and (iii) (Section 4.1)	161
A5. Statement of Birch's Regularity Conditions and Proof that the Minimum Power-Divergence Estimator Is BAN (Section 4.1)	163
A6. Proof of Results (i*), (ii*), and (iii*) (Section 4.1)	167
A7. The Power-Divergence Generalization of the Chernoff-Lehmann Statistic: An Outline (Section 4.1)	170
A8. Derivation of the Asymptotic Noncentral Chi-Squared Distribution for the Power-Divergence Statistic under Local Alternative Models (Section 4.2)	171
A9. Derivation of the Mean and Variance of the Power-Divergence Statistic for $\lambda > -1$ under a Nonlocal Alternative Model (Section 4.2)	172
A10. Proof of the Asymptotic Normality of the Power-Divergence Statistic under Sparseness Assumptions (Section 4.3)	174
A11. Derivation of the First Three Moments (to Order $1/n$ ) of the Power-Divergence Statistic for $\lambda > -1$ under the Classical (Fixed-Cells) Assumptions (Section 5.1)	175
A12. Derivation of the Second-Order Terms for the Distribution Function of the Power-Divergence Statistic under the Classical (Fixed-Cells) Assumptions (Section 5.2)	181
A13. Derivation of the Minimum Asymptotic Value of the Power-Divergence Statistic (Section 6.2)	183
A14. Limiting Form of the Power-Divergence Statistic as the Parameter $\lambda \rightarrow \pm\infty$ (Section 6.2)	183
 Bibliography	 185
Author Index	199
Subject Index	205