

Contents

Preface	xi
Chapter 1. Introduction	1
1.1. Modeling by stochastic differential equations	1
Chapter 2. Framework	11
2.1. White noise	11
The 1-dimensional, d -parameter smoothed white noise	
The (smoothed) white noise vector	
2.2. The Wiener–Itô chaos expansion	18
Chaos expansion in terms of Hermite polynomials	
Chaos expansion in terms of multiple Itô integrals	
2.3. Stochastic test functions and stochastic distributions.	
The Kondratiev spaces $(\mathcal{S})_{\rho}^{m:N}, (\mathcal{S})_{-\rho}^{m:N}$	28
The Hida test function space (\mathcal{S}) and the Hida distribution space $(\mathcal{S})^*$	
Singular white noise	
2.4. The Wick product	39
Some examples and counterexamples	
2.5. Wick multiplication and Itô/Skorohod integration	45
2.6. The Hermite transform	56
The characterization theorem for $(\mathcal{S})_1^N$	
Positive noise	
The positive noise matrix	
2.7. The $(\mathcal{S})_{\rho,r}^N$ spaces and the \mathcal{S} -transform	68
The \mathcal{S} -transform	
2.8. The topology of $(\mathcal{S})_{-1}^N$	74
Stochastic distribution processes	
2.9. The \mathcal{F} -transform and the Wick product on $L^1(\mu)$	80
Functional processes	
2.10. The Wick product and translation	84
2.11. Positivity	89
Exercises	96

Chapter 3. Applications to stochastic ordinary differential equations	105
3.1. Linear equations	105
Linear 1-dimensional equations	
Some remarks on numerical simulations	
Some linear multi-dimensional equations	
3.2. A model for population growth in a crowded stochastic environment	110
The general $(\mathcal{S})_{-1}$ solution	
A solution in $L^1(\mu)$	
A comparison of Model A and Model B.	
3.3. A general existence and uniqueness theorem	117
3.4. The stochastic Volterra equation	120
3.5. Wick products versus ordinary products: A comparison experiment .	128
Variance properties	
3.6. Solution and Wick approximation of quasilinear SDE	133
Exercises	138
Chapter 4. Stochastic partial differential equations	141
4.1. General remarks	141
4.2. The stochastic Poisson equation	143
The functional process approach	
4.3. The stochastic transport equation	146
Pollution in a turbulent medium	
The heat equation with a stochastic potential	
4.4. The stochastic Schrödinger equation	150
$L^1(\mu)$ -properties of the solution	
4.5. The viscous Burgers' equation with a stochastic source	159
4.6. The stochastic pressure equation	166
The smoothed positive noise case	
An inductive approximation procedure	
The 1-dimensional case	
The singular positive case	
4.7. The heat equation in a stochastic, anisotropic medium	175
4.8. A class of quasilinear parabolic SPDEs	179
4.9. SPDEs driven by Poissonian noise	182
Exercises	189

Appendix A. The Bochner–Minlos theorem	193
Appendix B. A brief review of Itô calculus	199
The Itô formula	
Stochastic differential equations	
The Girsanov theorem	
Appendix C. Properties of Hermite polynomials	207
Appendix D. Independence of bases in Wick products	298
References	217
List of frequently used notation and symbols	223
Index	227