

Contents

Preface	vii
I Fundamentals of MDS	1
1 The Four Purposes of Multidimensional Scaling	3
1.1 MDS as an Exploratory Technique	4
1.2 MDS for Testing Structural Hypotheses	6
1.3 MDS for Exploring Psychological Structures	9
1.4 MDS as a Model of Similarity Judgments	10
1.5 The Different Roots of MDS	13
2 Constructing MDS Representations	15
2.1 Constructing Ratio MDS Solutions	15
2.2 Constructing Ordinal MDS Solutions	19
2.3 Comparing Ordinal and Ratio MDS Solutions	25
2.4 On Flat and Curved Geometries	26
3 MDS Models and Measures of Fit	29
3.1 Basics of MDS Models	29
3.2 Errors, Loss Functions, and Stress	33
3.3 Stress Diagrams	34
3.4 Evaluating Stress	36
3.5 Recovering True Distances by Metric MDS	45

3.6	Further Variants of MDS Models	47
4	Three Applications of MDS	49
4.1	The Circular Structure of Color Similarities	49
4.2	The Regionality of Morse Codes Confusions	54
4.3	Dimensions of Facial Expressions	60
4.4	General Principles of Interpreting MDS Solutions	68
5	MDS and Facet Theory	71
5.1	Facets and Regions in MDS Space	71
5.2	Regional Laws	76
5.3	Multiple Facetizations	77
5.4	Partitioning MDS Spaces Using Facet Diagrams	79
5.5	Prototypical Roles of Facets	83
5.6	Criteria for Choosing Regions	84
5.7	Regions and Theory Construction	86
5.8	Regions, Clusters, and Factors	87
6	How to Obtain Proximities	91
6.1	Types of Proximities	91
6.2	Collecting Direct Proximities	92
6.3	Deriving Proximities by Aggregating over Other Measures	99
6.4	Proximities from Converting Other Measures	101
6.5	Proximities from Co-occurrence Data	103
6.6	Choosing a Particular Proximity	104
II	MDS Models and Solving MDS Problems	107
7	Matrix Algebra for MDS	109
7.1	Elementary Matrix Operations	109
7.2	Scalar Functions of Vectors and Matrices	112
7.3	Computing Distances Using Matrix Algebra	116
7.4	Eigendecompositions	117
7.5	Singular Value Decompositions	122
7.6	Some Further Remarks on SVD	124
7.7	Linear Equation Systems	125
7.8	Computing the Eigendecomposition	128
7.9	Configurations that Represent Scalar Products	131
7.10	Rotations	132
8	A Majorization Algorithm for Solving MDS	135
8.1	The Stress Function for MDS	135
8.2	Mathematical Excursus: Differentiation	137
8.3	Partial Derivatives and Matrix Traces	142

8.4	Minimizing a Function by Iterative Majorization	144
8.5	Majorizing Stress	150
9	Metric and Nonmetric MDS	159
9.1	Allowing for Transformations of the Proximities	159
9.2	Monotone Regression	165
9.3	The Geometry of Monotone Regression	169
9.4	Tied Data in Ordinal MDS	170
9.5	Rank-Images	172
9.6	Monotone Splines	173
10	Confirmatory MDS	181
10.1	Blind Loss Functions	181
10.2	Theory-Compatible MDS: An Example	182
10.3	Imposing External Constraints on MDS Representations	184
10.4	Weakly Constrained MDS	190
10.5	General Comments on Confirmatory MDS	195
11	MDS Fit Measures, Their Relations, and Some Algorithms	199
11.1	Normalized Stress and Raw Stress	199
11.2	Other Fit Measures and Recent Algorithms	202
12	Classical Scaling	207
12.1	Finding Coordinates in Classical Scaling	207
12.2	A Numerical Example for Classical Scaling	209
12.3	Choosing a Different Origin	210
12.4	Advanced Topics	211
13	Special Solutions, Degeneracies, and Local Minima	213
13.1	Special Solutions: Almost Equal Dissimilarities	213
13.2	A Degenerate Solution in Ordinal MDS	216
13.3	Avoiding Degenerate Solutions	219
13.4	Local Minima	220
13.5	Unidimensional Scaling	222
13.6	Full-Dimensional Scaling	226
13.7	The Tunneling Method for Avoiding Local Minima	227
III	Unfolding	229
14	Unfolding	231
14.1	The Ideal-Point Model	232
14.2	A Majorizing Algorithm for Unfolding	236
14.3	Unconditional Versus Conditional Unfolding	237
14.4	Trivial Unfolding Solutions and σ_2	239

14.5	Isotonic Regions and Indeterminacies	242
14.6	Unfolding Degeneracies in Practice and Metric Unfolding	245
14.7	An Ordinal-Interval Approach to Unfolding	249
14.8	Dimensions in Multidimensional Unfolding	250
14.9	Multiple Versus Multidimensional Unfolding	251
15	Special Unfolding Models	253
15.1	External Unfolding	253
15.2	The Vector Model of Unfolding	254
15.3	Weighted Unfolding	259
15.4	Value Scales and Distances in Unfolding	263
IV	MDS Geometry as a Substantive Model	271
16	MDS as a Psychological Model	273
16.1	Physical and Psychological Space	273
16.2	Minkowski Distances	277
16.3	Identifying the True Minkowski Distance	281
16.4	The Psychology of Rectangles	286
16.5	Axiomatic Foundations of Minkowski Spaces	291
16.6	Subadditivity and the MBR Metric	295
16.7	Minkowski Spaces, Metric Spaces, and Psychological Models	298
17	Scalar Products and Euclidean Distances	301
17.1	The Scalar Product Function	301
17.2	Collecting Scalar Products Empirically	304
17.3	Scalar Products and Euclidean Distances: Formal Relations	309
17.4	Scalar Products and Euclidean Distances: Empirical Relations	312
17.5	MDS of Scalar Products	315
18	Euclidean Embeddings	321
18.1	Distances and Euclidean Distances	321
18.2	Mapping Proximities into Distances	325
18.3	Maximal Dimensionality for Perfect Interval MDS	328
18.4	Mapping Fallible Proximities into Euclidean Distances	329
18.5	Fitting Proximities into a Euclidean Space	334
V	MDS and Related Methods	337
19	Procrustes Procedures	339
19.1	The Problem	339
19.2	Solving the Orthogonal Procrustean Problem	340
19.3	Examples for Orthogonal Procrustean Transformations	342
19.4	Procrustean Similarity Transformations	344

19.5	An Example of Procrustean Similarity Transformations . . .	346
19.6	Measuring Configurational Similarity by the Correlation Coefficient	347
19.7	Measuring Configurational Similarity by the Congruence Coefficient	349
19.8	Artificial Target Matrices in Procrustean Analysis	351
19.9	Other Generalizations of Procrustean Analysis	354
20	Three-Way Procrustean Models	357
20.1	Generalized Procrustean Analysis	357
20.2	Helm's Color Data	359
20.3	Generalized Procrustean Analysis	362
20.4	Individual Differences Models: Dimension Weights	365
20.5	An Application of the Dimension-Weighting Model	370
20.6	Vector Weightings	374
20.7	PINDIS, a Collection of Procrustean Models	378
21	Three-Way MDS Models	381
21.1	The Model: Individual Weights on Fixed Dimensions	381
21.2	The Generalized Euclidean Model	387
21.3	Some Algebra of Dimension-Weighting Models	390
21.4	Conditional and Unconditional Approaches	394
21.5	On the Dimension-Weighting Models	395
22	Methods Related to MDS	399
22.1	Principal Component Analysis	399
22.2	Models for Asymmetric Data	402
22.3	Correspondence Analysis	408
VI	Appendices	417
A	Computer Programs for MDS	419
B	Notation	435
	References	437
	Author Index	457
	Subject Index	463