

Contents

PREFACE	vii
CHAPTER 1 INTRODUCTION AND SUMMARY	
1.1 Three important practical problems	1
1.1.1 Forecasting time series	1
1.1.2 Estimation of transfer functions	2
1.1.3 Design of discrete control systems	4
1.2 Stochastic and deterministic dynamic mathematical models	7
1.2.1 Stationary and nonstationary stochastic models for forecasting and control	7
1.2.2 Transfer function models	13
1.2.3 Models for discrete control systems	16
1.3 Basic ideas in model building	17
1.3.1 Parsimony	17
1.3.2 Iterative stages in the selection of a model	18
PART I STOCHASTIC MODELS AND THEIR FORECASTING	
CHAPTER 2 THE AUTOCORRELATION FUNCTION AND SPECTRUM	
2.1 Autocorrelation properties of stationary models	23
2.1.1 Time series and stochastic processes	23
2.1.2 Stationary stochastic processes	26
2.1.3 Positive definiteness and the autocovariance matrix	28
2.1.4 The autocovariance and autocorrelation functions	30
2.1.5 Estimation of autocovariance and autocorrelation functions	32
2.1.6 Standard errors of autocorrelation estimates	34

2.2	Spectral properties of stationary models	36
2.2.1	The periodogram	36
2.2.2	Analysis of variance	37
2.2.3	The spectrum and spectral density function	39
2.2.4	Simple examples of autocorrelation and spectral density functions	42
2.2.5	Advantages and disadvantages of the autocorrelation and spectral density functions	44
A2.1	Link between the sample spectrum and autocovariance function estimate	44

CHAPTER 3 LINEAR STATIONARY MODELS

3.1	The general linear process	46
3.1.1	Two equivalent forms for the linear process	46
3.1.2	Autocovariance generating function of a linear process	48
3.1.3	Stationarity and invertibility conditions for a linear process	49
3.1.4	Autoregressive and moving average processes	51
3.2	Autoregressive processes	53
3.2.1	Stationarity conditions for autoregressive processes	53
3.2.2	Autocorrelation function and spectrum of autoregressive processes	54
3.2.3	The first order autoregressive (Markov) process	56
3.2.4	The second order autoregressive process	58
3.2.5	The partial autocorrelation function	64
3.2.6	Estimation of the partial autocorrelation function	65
3.2.7	Standard errors of partial autocorrelation estimates	65
3.3	Moving average processes	67
3.3.1	Invertibility conditions for moving average processes	67
3.3.2	Autocorrelation function and spectrum of moving average processes	68
3.3.3	The first-order moving average process	69
3.3.4	The second-order moving average process	
3.3.5	Duality between autoregressive and moving average processes	72
3.4	Mixed autoregressive—moving average processes	73
3.4.1	Stationarity and invertibility properties	73
3.4.2	Autocorrelation function and spectrum of mixed processes	74
3.4.3	The first-order autoregressive—first order moving average process	76
3.4.4	Summary	80
A3.1	Autocovariances, autocovariance generating functions and stationarity conditions for a general linear process	80

A3.2	A recursive method for calculating autoregressive parameters	82
CHAPTER 4 LINEAR NONSTATIONARY MODELS		
4.1	Autoregressive integrated moving average processes	85
4.1.1	The nonstationary first order autoregressive process	85
4.1.2	A general model for a nonstationary process exhibiting homogeneity	87
4.1.3	The general form of the autoregressive integrated moving average process	91
4.2	Three explicit forms for the autoregressive integrated moving average model	94
4.2.1	Difference equation form of the model	94
4.2.2	Random shock form of the model	95
4.2.3	Inverted form of the model	101
4.3	Integrated moving average processes	103
4.3.1	The integrated moving average process of order (0, 1, 1)	105
4.3.2	The integrated moving average process of order (0, 2, 2)	108
4.3.3	The general integrated moving average process of order (0, d , q)	112
A4.1	Linear difference equations	114
A4.2	The IMA (0, 1, 1) process with deterministic drift	119
A4.3	Properties of the finite summation operator	120
A4.4	ARIMA processes with added noise	121
A4.4.1	The sum of two independent moving average processes	121
A4.4.2	Effect of added noise on the general model	121
A4.4.3	Example for an IMA (0, 1, 1) process with added white noise	122
A4.4.4	Relation between the IMA (0, 1, 1) process and a random walk	123
A4.4.5	Autocovariance function of the general model with added correlated noise	124
CHAPTER 5 FORECASTING		
5.1	Minimum mean square error forecasts and their properties	126
5.1.1	Derivation of the minimum mean square error forecasts	127
5.1.2	Three basic forms for the forecast	129
5.2	Calculating and updating forecasts	132
5.2.1	A convenient format for the forecasts	132
5.2.2	Calculation of the ψ weights	132
5.2.3	Use of the ψ weights in updating the forecasts	134
5.2.4	Calculation of the probability limits of the forecasts at any lead time	135

5.3	The forecast function and forecast weights	138
5.3.1	The eventual forecast function determined by the autoregressive operator	139
5.3.2	Role of the moving average operator in fixing the initial values	139
5.3.3	The lead- l forecast weights	141
5.4	Examples of forecast functions and their updating	144
5.4.1	Forecasting an IMA (0, 1, 1) process	144
5.4.2	Forecasting an IMA (0, 2, 2) process	146
5.4.3	Forecasting a general IMA (0, d , q) process	149
5.4.4	Forecasting autoregressive processes	150
5.4.5	Forecasting a (1, 0, 1) process	152
5.4.6	Forecasting a (1, 1, 1) process	154
5.5	Summary	155
A5.1	Correlations between forecast errors	158
A5.1.1	Autocorrelation function of forecast errors at different origins	158
A5.1.2	Correlation between forecast errors at the same origin with different lead times	159
A5.2	Forecast weights for any lead time	160
A5.3	Forecasting in terms of the general integrated form	162
A5.3.1	A general method of obtaining the integrated form	162
A5.3.2	Updating the general integrated form	164
A5.3.3	Comparison with the discounted least squares method of R. G. Brown	166

PART II STOCHASTIC MODEL BUILDING

CHAPTER 6 MODEL IDENTIFICATION

6.1	Objectives of identification	173
6.1.1	Stages in the identification procedure	173
6.2	Identification techniques	174
6.2.1	Use of the autocorrelation and partial autocorrelation functions in identification	174
6.2.2	Standard errors for estimated autocorrelations and partial autocorrelations	177
6.2.3	Identification of some actual time series	178
6.3	Initial estimates for the parameters	187
6.3.1	Uniqueness of estimates obtained from the autocovariance function	187
6.3.2	Initial estimates for moving average processes	187
6.3.3	Initial estimates for autoregressive processes	189

6.3.4	Initial estimates for mixed autoregressive-moving average processes	190
6.3.5	Choice between stationary and nonstationary models in doubtful cases	192
6.3.6	Initial estimate of residual variance	193
6.3.7	An approximate standard error for \bar{w}	193
6.4	Model multiplicity	195
6.4.1	Multiplicity of autoregressive-moving average models	195
6.4.2	Multiple moment solutions for moving average parameters	198
6.4.3	Use of the backward process to determine starting values	199
A6.1	The expected value of the estimated autocorrelation function for a nonstationary process	200
A6.2	A general method for obtaining initial estimates of the parameters of a mixed autoregressive-moving average process	201
A6.3	The forward and backward IMA processes of order (0, 1, 1).	205

CHAPTER 7 MODEL ESTIMATION

7.1	Study of the likelihood and sum of squares functions	208
7.1.1	The likelihood function	208
7.1.2	The conditional likelihood for an ARIMA process	209
7.1.3	Choice of starting values for conditional calculation	210
7.1.4	The unconditional likelihood—the sum of squares function—least squares estimates	212
7.1.5	General procedure for calculating the unconditional sum of squares	215
7.1.6	Graphical study of the sum of squares function	220
7.1.7	Description of “well-behaved” estimation situations—confidence regions	224
7.2	Nonlinear estimation	231
7.2.1	General method of approach	231
7.2.2	Numerical estimates of the derivatives	233
7.2.3	Direct evaluation of the derivatives	235
7.2.4	A general least squares algorithm for the conditional model	236
7.2.5	Summary of models fitted to Series A–F	238
7.2.6	Large sample information matrices and covariance estimates	240
7.3	Some estimation results for specific models	243
7.3.1	Autoregressive processes	243
7.3.2	Moving average processes	245
7.3.3	Mixed processes	245
7.3.4	Separation of linear and nonlinear components in estimation	246

7.3.5	Parameter redundancy	248
7.4	Estimation using Bayes' theorem	250
7.4.1	Bayes' theorem	250
7.4.2	Bayesian estimation of parameters	252
7.4.3	Autoregressive processes	253
7.4.4	Moving average processes	255
7.4.5	Mixed processes	257
A7.1	Review of normal distribution theory	258
A7.2	A review of linear least squares theory	265
A7.3	Examples of the effect of parameter estimation errors on probability limits for forecasts	267
A7.4	The exact likelihood function for a moving average process	269
A7.5	The exact likelihood function for an autoregressive process	274

CHAPTER 8 MODEL DIAGNOSTIC CHECKING

8.1	Checking the stochastic model.	285
8.1.1	General philosophy	285
8.1.2	Overfitting	286
8.2	Diagnostic checks applied to residuals	287
8.2.1	Autocorrelation check	289
8.2.2	A portmanteau lack of fit test	290
8.2.3	Model inadequacy arising from changes in parameter values	293
8.2.4	Cumulative periodogram check	294
8.3	Use of residuals to modify the model	298
8.3.1	Nature of the correlations in the residuals when an incorrect model is used	298
8.3.2	Use of residuals to modify the model	299

CHAPTER 9 SEASONAL MODELS

9.1	Parsimonious models for seasonal time series	300
9.1.1	Fitting versus forecasting	301
9.1.2	Seasonal models involving adaptive sines and cosines	301
9.1.3	The general multiplicative seasonal model	303
9.2	Representation of the airline data by a multiplicative $(0, 1, 1) \times (0, 1, 1)_{12}$ seasonal model	305
9.2.1	The multiplicative $(0, 1, 1) \times (0, 1, 1)_{12}$ model	305
9.2.2	Forecasting	306
9.2.3	Identification	313
9.2.4	Estimation	315
9.2.5	Diagnostic checking	320
9.3	Some aspects of more general seasonal models	322
9.3.1	Multiplicative and nonmultiplicative models	322

9.3.2	Identification	324
9.3.3	Estimation	325
9.3.4	Eventual forecast functions for various seasonal models	325
A9.1	Autocovariances for some seasonal models	329

PART III TRANSFER FUNCTION MODEL BUILDING

CHAPTER 10 TRANSFER FUNCTION MODELS

10.1	Linear transfer function models	337
10.1.1	The discrete transfer function	338
10.1.2	Continuous dynamic models represented by differential equations	340
10.2	Discrete dynamic models represented by difference equations	345
10.2.1	The general form of the difference equation	345
10.2.2	Nature of the transfer function	346
10.2.3	First and second order discrete transfer function models	348
10.2.4	Recursive computation of output for any input	353
10.3	Relation between discrete and continuous models	355
10.3.1	Response to a pulsed input	356
10.3.2	Relationships for first and second order coincident systems	358
10.3.3	Approximating general continuous models by discrete models	361
10.3.4	Transfer function models with added noise	362
A10.1	Continuous models with pulsed inputs	363
A10.2	Nonlinear transfer functions and linearization	367

CHAPTER 11 IDENTIFICATION, FITTING, AND CHECKING OF TRANSFER FUNCTION MODELS

11.1	The cross correlation function	371
11.1.1	Properties of the cross covariance and cross correlation functions	371
11.1.2	Estimation of the cross covariance and cross correlation functions	374
11.1.3	Approximate standard errors of cross correlation estimates	376
11.2	Identification of transfer function models	377
11.2.1	Identification of transfer function models by prewhitening the input	379
11.2.2	An example of the identification of a transfer function model	381
11.2.3	Identification of the noise model	383
11.2.4	Some general considerations in identifying transfer function models	386

11.3	Fitting and checking transfer function models	388
11.3.1	The conditional sum of squares function	388
11.3.2	Nonlinear estimation	390
11.3.3	Use of residuals for diagnostic checking	392
11.3.4	Specific checks applied to the residuals	393
11.4	Some examples of fitting and checking transfer function models	395
11.4.1	Fitting and checking of the gas furnace model	395
11.4.2	A simulated example with two inputs	400
11.5	Forecasting using leading indicators	402
11.5.1	The minimum square error forecast	404
11.5.2	Forecast of CO ₂ output from gas furnace	407
11.5.3	Forecast of nonstationary sales data using a leading indicator	409
11.6	Some aspects of the design of experiments to estimate transfer functions	412
A11.1	Use of cross spectral analysis for transfer function model identification	413
A11.1.1	Identification of single input transfer function models	413
A11.1.2	Identification of multiple input transfer function models	415
A11.2	Choice of input to provide optimal parameter estimates	416
A11.2.1	Design of optimal input for a simple system	416
A11.2.2	A numerical example	418

PART IV DESIGN OF DISCRETE CONTROL SCHEMES

CHAPTER 12 DESIGN OF FEEDFORWARD AND FEEDBACK CONTROL SCHEMES

12.1	Feedforward control	423
12.1.1	Feedforward control to minimize mean square error at the output	424
12.1.2	An example—control of specific gravity of an intermediate product	427
12.1.3	A nomogram for feedforward control	430
12.1.4	Feedforward control with multiple inputs	432
12.2	Feedback control	433
12.2.1	Feedback control to minimize output mean square error	434
12.2.2	Application of the control equation: relation with three-term controller	436
12.2.3	Examples of discrete feedback control	437

12.3	Feedforward-feedback control	446
12.3.1	Feedforward-feedback control to minimize output mean square error	448
12.3.2	An example of feedforward-feedback control	448
12.3.3	Advantages and disadvantages of feedforward and feedback control	449
12.4	Fitting transfer function-noise models using operating data	451
12.4.1	Iterative model building	451
12.4.2	Estimation from operating data	451
12.4.3	An example	453
 CHAPTER 13 SOME FURTHER PROBLEMS IN CONTROL		
13.1	Effect of added noise in feedback schemes	460
13.1.1	Effect of ignoring added noise—rounded schemes	461
13.1.2	Optimal action when there are observational errors in the adjustments x_t	465
13.1.3	Transference of the noise origin	469
13.2	Feedback control schemes where the adjustment variance is restricted	472
13.2.1	Derivation of optimal adjustment	474
13.2.2	A constrained scheme for the viscosity/gas rate example	483
13.3	Choice of the sampling interval	486
13.3.1	An illustration of the effect of reducing sampling frequency	487
13.3.2	Sampling an IMA (0, 1, 1) process	488
 PART V		
	Description of computer programs	495
	Collection of tables and charts	517
	Collection of time series used for examples in the text	524
	References	538
	Index	543