TABLE OF CONTENTS

Page

Preface iii

Preface to the American Edition v

Introduction 1

Chapter I. The multiple Laplace transformation, functions of several complex variables, and analytic sheaves 7

§1. The multiple Laplace transformation 7

§2. Functions of several complex variables. Theorems yielding bounds 10

§3. Ideals in rings of holomorphic functions. Analytic sheaves 17

Chapter II. Sufficient statistics and exponential families 23

§1. General information on sufficient statistics 23

§2. Examples of sufficient statistics 27

§3. Informational properties of sufficient statistics 32

§4. Sufficient statistics for a repeated sample. Exponential families 35

§5. Exponential families 41

§6. Sufficient statistics and unbiased estimates 45

Chapter III. Nuisance parameters. Tests with invariant power functions 49

§1. Nuisance parameters 49

§2. Tests with invariant power functions 51

§3. Some results dealing with tests with invariant power functions 53

§4. Stein's test 59

Chapter IV. Similar tests and statistics 63

§1. Similarity of tests and of statistics 63

§2. Neyman structures. Lehmann's and Scheffé's theorems 66

§3. Some methods of constructing similar zones 69

§4. Approximately similar zones 78

§5. Independent statistics 80

§6. Applications of a theorem of H. Cartan to the study of families of statistics 83

Chapter V. Cotest ideals for exponential families 89

§1. Similar tests and cotest ideals 89
§2. Statement of the problem for incomplete exponential families 92
§3. Ideals of precotests 94
§4. Application of Cartan's theorems 97
§5. The behavior of smooth precotests 98
§6. Smoothing of precotests 101
§7. Formation of smooth precotests from a given one 104
§8. Formulation of the final results. Examples 106

Chapter VI. Wijsman's D-method 117

§1. The D-method and the conditions under which it can be applied 117
§2. Examples of application of the D-method 119

Chapter VII. Unbiased estimates 125

§1. Unbiased estimates for incomplete exponential families depending on sufficient statistics 125
§2. On the behavior of the variance of unbiased estimates 127
§3. A theorem of S. R. Rao on the inadmissibility of certain estimates 133

Chapter VIII. Analytical methods of studying unrandomized tests. Application to the Behrens-Fisher problem 139

§1. Questions of existence of unrandomized similar tests for incomplete exponential families 139
§2. Statement of the problem of an unrandomized homogeneous similar test in the Behrens-Fisher problem 142
§3. Homogeneous Fisher-Welch-Wald tests 146
§4. Lemmas on tangency of a test boundary to a critic 151
§5. Completion of the proof of theorem 8.3.1. 166

Chapter IX. Randomized homogeneous tests in the Behrens-Fisher problem. Characterization of tests of the Bartlett-Scheffé type 167

§1. Nonexistence of "null-regular" similar tests 167
§2. Bartlett-Scheffé tests 178
§3. A homogeneous randomized test associated with Bartlett's test 181
§4. Characterization of tests of the Bartlett-Scheffé type 183

Chapter X. An unrandomized homogeneous similar test in the Behrens-Fisher problem 193

§1. Construction of an unrandomized test 193

§2. The Romanovskiĭ-Sudakov theorem 197

Chapter XI. The problem of many small samples 204

§1. Statement of the problem 204

§2. A. A. Petrov's investigations 205

Appendix 213

Supplement. New results in the theory of estimation and testing hypotheses for problems with nuisance parameters 217

§1. Invariant verification of functions which are polynomials in a and $1/\sigma^2$ for normal samples 217

§2. The description of all cotests for a class of exponential families with polynomial relations 226

§3. Conditions of optimal unbiased estimation for incomplete exponential families with polynomial relations 232

§4. The sample mean as the estimate of scale parameters 240

§5. Nonparametric approach to the estimation of location parameters 243

Bibliography to the Appendix 249

Bibliography 251