

# Contents

LIST OF CONTRIBUTORS	v
PREFACE	ix
<b>Opening Remarks</b>	1
<i>Ernest A. Pinson</i>	

## PART I / **Nonparametric Methods**

### **Some Nonparametric Multivariate Procedures Based on Statistically Equivalent Blocks**

*T. W. Anderson*

1. Introduction	5
2. Statistically Equivalent Blocks	6
3. Testing the Hypothesis that an Unknown Cumulative Distribution Is a Specified One	11
4. Testing the Hypothesis that Two Unknown Distributions Are Identical	18
5. Classification of an Observation into One of Two Populations	23
References	27

### **Some Nonparametric Tests for the Multivariate Several Sample Location Problem**

*V. P. Bhapkar*

1. Summary	29
2. Introduction and Notation	29
3. The Asymptotic Distribution of a Test Criterion under $H_0$	31
4. Special Cases	37
5. Remarks	40
References	40

### **Nonparametric Estimation of Multivariate Densities with Applications**

*V. K. Murthy*

Summary	43
1. Summary of Results in the One-Dimensional Case	43
2. Multivariate Case	45

3. A Class of Estimators for the Multivariate Density at a Point of Continuity of the Distribution $F(x_1, x_2, \dots, x_p)$ and the Density $f(x_1, x_2, \dots, x_p)$	50
4. Asymptotic Normality of the Sequence $\{f_n(x_1, x_2, \dots, x_p)\}$ at a Point of Continuity of $F(x_1, x_2, \dots, x_p)$ and $f(x_1, x_2, \dots, x_p)$	52
5. Multidimensional Windows	54
6. Applications to Reliability and Hazard Rate	54
References	56

## PART II / Multivariate Analysis of Variance and Related Topics

### A $k$ -Sample Regression Model with Covariance

*Leon Gleser and Ingram Olkin*

1. Introduction and Summary	59
2. A Canonical Form	61
3. The Likelihood-Ratio Test for Testing the Equality of the $\beta^{(i)}$	63
4. The Distribution of the Maximum-Likelihood Estimators	69
References	72

### Some Multivariate Tests with Restricted Alternative Hypotheses

*A. Kudô and H. Fujisawa*

1. A Bivariate Analogue of a Two-Sided Test	73
2. The Power Function	75
3. Numerical Results and Discussions	77
4. An Alternative Test	80
5. The Scope of Generalizations	81
References	84

### Covariance Adjustment and Related Problems in Multivariate Analysis

*C. Radhakrishna Rao*

1. Summary	87
2. Discrimination between Two Populations	88
3. Discriminant Functions	90
4. The Effect of Increasing the Number of Characters	91
5. Analysis of Dispersion with Covariance Adjustment	92
6. The Model of Potthoff and Roy	99
7. An Illustrative Example	100
References	102

**Power of the Likelihood-Ratio Test Used in Analysis of Dispersion***J. Roy*

1. Summary	105
2. Notations	105
3. General Multivariate Linear Hypothesis	106
4. Test Procedures	107
5. The Nonnull Distribution of Wilks' Criterion	109
6. The Exact Power Function of Wilks' Test when $p = 2$ and $t = 1$	110
7. Approximations Using Jacobi Polynomials	112
8. Tables	114
References	126

**Some Generalizations of Multivariate Analysis of Variance***J. N. Srivastava*

1. Summary	129
2. Introduction	129
3. Standard MANOVA	130
4. Patterns of Heterogeneity Different for Different Responses: Multivariate Block Designs	132
5. Responses with Unequal Degrees of Importance: Hierarchical Multiresponse Designs	133
6. Pooling of Multiresponse Experiments: General Incomplete Multiresponse Designs	134
7. MANOVA Inapplicable, Although Each Response Observed on Each Unit	138
8. Multiresponse Surfaces	140
References	144

**PART III / Classification****Predictive Discrimination***Seymour Geisser*

Introduction	149
1. General Outline	150
2. Multivariate Normal Discrimination	153
3. Joint Normal Classification	157
4. Sequential Normal Classification	159
5. Remarks	161
References	163

**Discrimination and Classification***M. G. Kendall*

1. Introduction	165
2. Discrimination	166
3. Convex-Hull Method	168
4. The Order-Statistic Method	171
5. Classification	180
6. Summary	184
References	184

**A Distance and Related Statistics in Multivariate Analysis***Kameo Matusita*

1. Introduction	187
2. The Distance Method	187
3. A Distance and a Related Quantity	189
4. Multivariate Gaussian Case	190
5. "Test" Statistics	192
6. Distribution of $\rho$ , $\rho_1$ , $\rho_2$ , and $\rho_3$	193
7. Illustration	198
References	200

**PART IV / Distribution Theory****A Multiple Behrens-Fisher Distribution***E. A. Cornish*

1. Recapitulation of the Univariate Case	203
2. A Multivariate Distribution	204
3. Different Residual Variances in the Same Experiment	206
References	207

**Inference on Latent Roots by Calculation of Hypergeometric Functions of Matrix Argument***Alan T. James*

1. Summary	209
2. A Brief Survey of Likelihood Functions of Canonical Analysis	210
3. Hypergeometric Functions of Matrix Argument and Zonal Polynomials	211
4. Principal Components	213
5. Ratios of the Roots	216

6. The Bivariate Case	217
7. Asymptotic Formula for ${}_0F_0^{(m)}$	222
8. Maximum Likelihood and Bias	223
9. Combined Asymptotic and Power-Series Expansion	226
10. Derivation of the Combined Asymptotic and Power-Series Approximation for $m = 3$	228
11. Influence of Distant Roots on the Likelihood of the Ratio of Adjacent Roots	231
12. Numerical Example	231
References	234

## Noncentral Multivariate Beta Distribution and the Moments of Traces of Some Matrices

*K. C. Sreedharan Pillai*

1. Introduction and Summary	237
2. Independent Beta Variables	238
3. Traces of Some Matrices as Functions of Independent Beta Variables	241
4. Moments of $W^{(2)}$ , $V^{(2)}$ , and $U^{(2)}$	242
5. Approximations to the Distributions of $W^{(2)}$ , $V^{(2)}$ , and $U^{(2)}$	244
6. Power Functions of Tests of Hypothesis: $\lambda = 0$ against $\lambda > 0$ Based on $V^{(2)}$ , $U^{(2)}$ , and $\Lambda$	247
7. Power Functions for Tests of Hypothesis: $\rho = 0$ against $\rho > 0$ Based on $V^{(2)}$ , $U^{(2)}$ , and $\Lambda$ , and the Largest Root	248
References	251

## PART V / Optimum Properties of Test Procedures

### Multivariate Optimality Results

*J. Kiefer*

1. Introduction; Invariance and Formal Bayes Procedures	255
2. Minimax Questions; The Hunt-Stein Theorem	261
3. Admissibility and Complete Classes	265
References	272

### Fully Invariant Proper Bayes Tests

*Richard Schwartz*

1. Introduction	275
2. Notation and the MANOVA Problem	276
3. Improper Bayes Tests	278
4. Proper Bayes Tests	280
5. Remarks	282
References	284

PART VI / **Estimation and Prediction****Multidimensional Information Inequalities and Prediction***M. H. DeGroot and M. M. Rao*

Introduction	287
1. Analysis of the Standard Regularity Conditions	288
2. Sequential Sampling Plans	294
3. Efficient Estimation in the Wide Sense	298
4. Multidimensional Bayes Estimation	299
5. Multidimensional Prediction Theory	306
References	311

**Estimation in Multivariate Analysis***A. P. Dempster*

1. Introduction	315
2. A Laboratory for Comparing Methodologies	315
3. The Modified Jackknife Applied to Canonical Correlation Analysis	321
References	333

**On the Fix-Point Property of Wold's Iterative Estimation Method for Principal Components***Ejnar Lyttkens*

1. The Method	335
2. The Relation to Other Procedures	339
3. A Geometrical Interpretation of the Iteration	340
4. The Fix-Point Property	342
5. The Fix-Point Property in the Sample Space	346
6. A Detailed Comparison between the Procedures	348
7. Note on the Numerical Accuracy of Wold's Method	349
References	350

**Recent Trends in Multivariate Prediction Theory***P. Masani*

1. Introduction	351
2. The Gram Matricial Structure of $\mathcal{H}^q$	353
3. Elementary Solution of the Prediction Problem	355
4. The Shift Operator and Wold-Zasuhin Decomposition	356
5. Spectral Analysis	359
6. The Space $L_{2,\mathcal{M}}$ for a Nonnegative Hermitian Matrix-Valued Measure	361
7. Isomorphism between the Temporal and Spectral Spaces	363
8. Cross-Covariance and Spectra. Subordination	364

9. Spectral Analysis of a Purely Nondeterministic S. P.	367
10. Spectral Analysis of a Full-Rank S. P.	369
11. Concordance of Wold-Zasuhin and Lebesgue-Cramér Decompositions for Degenerate Ranks	370
12. Degenerate Rank Factorization	371
13. Spectral and Autoregressive Representations for the Predictor of a Purely Nondeterministic S. P.	373
14. Determination of the Generating Function from the Spectral Density	377
15. The Factorization of Matricial Rational Spectral Densities	378
References	381

## Remarks on Higher Order Spectra

*M. Rosenblatt*

1. Introduction	383
2. Basic Concepts	383
3. Bispectral Estimates	387
References	388

## Estimation of Principal Components and Related Models by Iterative Least Squares

*Herman Wold*

Summary	391
1. Component Structure vs. Factor Structure	391
2. Component Structure as a Basis for the Estimation of One-Factor Structure	402
3. Extensions of the Principal-Components Approach	417
References	419

## PART VII / Ranking and Selection Procedures

### An Inverse-Sampling Procedure for Selecting the Most Probable Event in a Multinomial Distribution

*Theophilos Cacoullos and Milton Sobel*

1. Summary	423
2. Introduction	424
3. Formulation of the Problem	425
4. The $P\{CS R_1\}$ and the Determination of $N$	425
5. Asymptotic Theory	427
6. Exact and Asymptotic Evaluation of $E(T)$	433
7. Comparison of ISP and FSP	444
Addenda	447
References	455

## On Some Selection and Ranking Procedures for Multivariate Normal Populations Using Distance Functions

*Shanti S. Gupta*

1. Introduction and Summary	457
2. Statement of the Problem	458
3. A Class of Selection and Ranking Procedures	460
4. Procedure $R$ and the Probability of Correct Selection	463
5. Distribution of an Associated Statistic	470
6. The Infimum of the Probability of Correct Selection for the Case $k = 2$	471
7. Moments of the Statistic $Z$	473
References	474

## Some Procedures for Selection of Multivariate Normal Populations Better than a Control

*P. R. Krishnaiah and M. Haseeb Rizvi*

1. Introduction and Summary	477
2. Notations and Preliminaries	478
3. Selection Based on Linear Combinations of the Elements of Mean Vector—One-Sided Case	479
4. Selection Based on Linear Combinations of the Elements of Mean Vector—Two-Sided Case	482
5. Selection Based on Absolute Values of the Linear Combinations of Elements of Mean Vector	483
6. Selection Based on Distances of Populations from Control Population	485
7. Selection Based on Distances of Populations from a Multivariate Normal Population with Zero Mean Vector	487
8. Concluding Remarks	490
References	490

## PART VIII / Applications

### Clustering Procedures

*J. J. Fortier and H. Solomon*

Introduction and Summary	493
1. The $B$ Clustering Function: An Unsuccessful Attempt	494
2. A Clustering Function Which Is Defined for Each Pair of Variables	499
3. A Generalization of the $C^*$ Criterion	504
References	506



## Sequential Multivariate Procedures for Means with Quality Control Applications

*J. Edward Jackson and Ralph A. Bradley*

1. Introduction	507
2. Sequential Multivariate Procedures for Testing Means	508
3. Two-Sample Cases	511
4. Generalized $\chi^2$ Statistics	511
5. Problems Associated with Specifying Hypotheses	511
6. Example	513
References	518

## Multivariate Responses in Comparative Experiments

*Oscar Kempthorne*

1. Introduction	521
2. Classes of Experiment	522
3. The Univariate Experiment	524
4. What Do Scientists Want from Multivariate Experiments?	528
5. Conclusions	537
References	539

## Equating of Grades or Scores on the Basis of a Common Battery of Measurements

*Richard F. Potthoff*

1. Summary and Introduction	541
2. Statement of the Problem; Notation	541
3. The Different Models	543
4. Appropriateness of the Different Models	546
5. Other Work Related to This Paper	547
6. Further Notation	548
7. The Maximum-Likelihood Estimates: Listing of Results	549
8. The Maximum-Likelihood Estimates: Derivations of the Results	553
References	558

## Parametric Representation of Nonlinear Data Structures

*Roger N. Shepard and J. Douglas Carroll*

Abstract	561
1. Introduction: The Problem	562
2. Proximity Analysis of Data Profiles	567
3. Parametric Mapping by Optimizing an Index of Continuity	578
References	591