

Contents

LIST OF CONTRIBUTORS

v

PREFACE

ix

PART I / **Nonparametric Methods**

Some Nonparametric Tests for the Multivariate Goodness-of-Fit, Multisample, Independence, and Symmetry Problems

C. B. Bell and Paul J. Smith

1. Introduction and Summary	3
2. Preliminaries and Notation	3
3. One-Sample Tests and Statistics	6
4. Multisample DF Procedures	9
5. Tests of Independence	14
6. Total Independence and Rank Procedures	17
7. Tests of Symmetry	19
8. Open Problems and Concluding Remarks	22
References	23

Conditional Probability Density and Regression Estimators

M. Rosenblatt

1. Introduction	25
2. Preliminaries	25
3. Asymptotic Behavior of the Estimators	27
References	31

On Robust Nonparametric Estimation in Some Multivariate Linear Models

Pranab Kumar Sen and Madan Lal Puri

1. Introduction	33
2. Formulation of the Problems	34
3. Preliminary Notions and Basic Assumptions	35

4. Point Estimation of α when It Is Assumed that $\beta = 0$	37
5. Estimation of the Regression Parameter β	39
6. Asymptotic Relative Efficiencies (ARE) of the Estimators	42
7. Joint Estimation of (α, β)	47
8. Confidence Regions for α and β	49
References	51

PART II / **Multivariate Analysis of Variance and Related Topics**

Statistical Inference for Covariance Matrices with Linear Structure

T. W. Anderson

1. Introduction	55
2. Maximum Likelihood Estimates of the Coefficients in the Covariance Matrix	57
3. Likelihood Ratio Criteria for the Hypothesis that the Covariance Matrix Is a Linear Combination	59
4. Maximum Likelihood Estimates and Markov Estimates	60
5. Maximum Likelihood Estimates of the Coefficients in the Inverse of the Covariance Matrix	62
6. Likelihood Ratio Criteria for the Hypothesis that the Inverse of a Covariance Matrix Is a Linear Combination	63
7. An Example	64
8. On Computation of Estimates	65
References	66

A Comparison of Some Methods of Simultaneous Inference in MANOVA

K. R. Gabriel

Introduction and Summary	67
1. Data and Notation	67
2. Model and Hypotheses	69
3. Available Procedures for Simultaneous Inference	71
4. Properties and Comparisons of Procedures	75
5. An Example	80
References	85

On the Effect of Heteroscedasticity and Nonnormality upon Some Multivariate Test Procedures

Koichi Ito

1. Introduction and Summary	87
2. Preliminaries—The Mathematical Model of k Samples	89

PART I. Testing Hypotheses about Mean Vectors	90
3. An Exact Test in the Normal, Heteroscedastic Case	91
4. Two Approximate Tests in the Normal, Heteroscedastic Case	98
5. Three Approximate Tests in the Nonnormal Case	104

PART II. Testing Hypotheses about Var-Covar Matrices

6. Four Approximate Tests in the Nonnormal Case
References

111
113
119

Simultaneous Test Procedures under General MANOVA Models*P. R. Krishnaiah*

1. Introduction
2. Notation and Preliminaries
3. Simultaneous Tests under Models of the Form $E(X) = A\mu B$
4. Simultaneous Test Procedures under MANOVA Models with Autocorrelated Errors
5. Simultaneous Tests under MANOVA Models with a General Correlation Pattern
References

121
122
124
134
139
141

Some Studies on Intersection Tests in Multivariate Analysis of Variance*J. N. Srivastava*

1. Summary
2. Introduction
3. Choices of Constants in Simultaneous but Independent F -Tests
4. Choice of Constants in the Simultaneous Analysis of Variance Test (Quasi-Independent Case)
5. Choice of Constants in the Simultaneous Analysis of Variance Test (Dependent Case)
6. Discussion
References

145
145
146
152
160
164
167

PART III / Distribution Theory**The Exact Distributions of Likelihood Criteria for Different Hypotheses***P. C. Consul*

1. Introduction
2. Exact Probability Distribution Function of L
3. Test of Independence of Two Sets of Variables
4. Testing Linear Hypotheses about Regression Coefficients
5. Test of Independence of Many Sets of Variates
6. Testing Sphericity and Symmetry
7. Testing Compound Symmetry
Appendix: Meijer's Function and Some Identities
References

171
172
173
174
175
176
178
180
180

What Is the Use of a Distribution?*I. J. Good*

Summary	183
Introduction	184
1. A Proposed Utility Measure	186
2. Infinite α	189
3. Application to the Distribution of Parameters	191
4. Minimax Utility and Invariant Initial Distributions	192
5. Finite α	193
6. Multinormal Example	194
7. Entropy, Dientropy, and Trientropy	197
8. Discussion and Generalization	199
References	202

Tests of Equality of Latent Roots of the Covariance Matrix*A. T. James*

1. Summary	205
2. Introduction	205
3. Asymptotic Approximation to the Roots Distribution	206
4. Test of Equality of the Last q Roots	210
5. Test of Equality of Two Latent Roots in the Trivariate Case	214
References	217

Distributions of Vectors Corresponding to the Largest Roots of Three Matrices*C. G. Khatri and K. C. S. Pillai*

1. Introduction and Summary	219
2. Notations and Some Useful Results	220
3. Distribution of CVCLR in the Single Sample Case	222
4. Distributions of CVCLR's in the Two Sample Case	227
5. Testing of Hypothetical Principal Vectors of Σ_1 in the field of Σ_2	236
6. Remarks	239
References	240

Some Identities and Approximations Concerning Positive and Negative Multinomial Distributions*C. G. Khatri and Sujit Kumar Mitra*

Summary	241
1. Introduction	241
2. Relation between Multinomial and Multivariate Beta Distributions	243
3. Useful Approximations to the Multinomial Distributions	248
4. Approximations to the CDF of a Compound Multinomial	258
References	260

Approximate Confidence Regions for Constraint Parameters*Albert Madansky and Ingram Olkin*

1. Introduction	261
2. A Method for Obtaining Approximate Confidence Regions	262
3. Preliminaries and Notation	267
4. One Sample Problem for the Wishart Distribution	269
5. Two Sample Problems for the Wishart Distribution	276
6. Confidence Interval for $\mu\Sigma^{-1}\mu'$	283
7. Behrens-Fisher Model	284
References	286

PART IV / Characteristic Functions and Characterization Problems**Characteristic Functions of Random Variables Connected with Brownian Motion and of the von Mises Multidimensional ω_n^2** *Daniel Dugue*

1. Multivariate Characteristic Functions	289
2. Multidimensional von Mises ω_n^2	295
3. Two Variables in Unidimensional Brownian Motion	298
References	301

Some Recent Developments in the Theory of Multivariate Characteristic Functions*Eugene Lukacs*

1. Introduction	303
2. Some Classical Results on Multivariate Characteristic Functions	304
3. Multivariate Analytic Characteristic Functions	309
4. Factorization Problems—the Class I_0	311
5. The α -Decompositions	314
6. Concluding Remarks	315
References	317

Some Characterizations of the Multivariate Normal Distribution*C. Radhakrishna Rao*

Summary	321
1. Introduction	321
2. Multivariate Extension of Kagan-Linnik-Rao Theorem	323
3. The Special Case of $n = 2$	324
4. Theorems on Variables with a Linear Structure	326
References	328

PART V / **Time Series and Stochastic Processes****The Canonical Analysis of Stationary Time Series***David R. Brillinger*

1. Summary	331
2. Introduction and Notation	331
3. Canonical Variates for Time Series	334
4. Statistical Properties	337
5. A Worked Example	342
6. Open Questions	343
7. Proofs	343
References	349

Theory of Time-Varying Spectral Analysis and Complex Wishart Matrix Processes*N. R. Goodman and M. R. Dubman*

Introduction and Summary	351
1. The Multivariate Complex Gaussian Distribution	352
2. The Central Complex Wishart Distribution and Related Distributions	354
3. The Noncentral Complex Wishart Distribution	354
4. The Joint Distribution of Two Correlated Central Complex Wishart Matrices (the Bivariate 2×2 Complex Wishart Distribution)	355
5. The Multivariate Complex Wishart Distribution	356
6. Complex Wishart Processes	357
7. A Particular Class of Complex Wishart Processes	358
8. Time-Varying Spectral Estimators	361
9. Distributional Results Pertaining to Time-Varying Spectral Estimators	362
References	365

Stochastic Differential Equations in Statistical Estimation Problems*G. Kallianpur and C. Striebel*

1. Introduction	367
2. A Bayes Formula for the Conditional Expectation $E[g \mathbf{z}(\tau), 0 \leq \tau \leq t]$	369
3. Stochastic Integrals and Differentials of Ito Type	375
4. Markov Processes with Extended Infinitesimal Generator	380
5. Ito Stochastic Differential Equation for $E^t[f(x(t))]$	382
6. The Linear Filter: The Kalman-Bucy Theory	384
References	388

Multiple Time Series Modeling*Emanuel Parzen*

1. Introduction	389
2. Innovation Approaches to Modeling	390
3. Spectral Approaches to Modeling	395
4. Relations between Time Series	398

5. Autoregressive Approach to a Single Time Series	402
6. Multiple Spectral Density Estimation	405
References	407

Representation Theory of Multidimensional Generalized Random Fields

M. M. Rao

Introduction	411
1. Preliminaries	412
2. Covariance Structure	416
3. Representation Theory	420
4. Local Classes of Random Fields	427
5. Isotropic and Harmonizable Fields	432
References	435

On a New Class of Estimates

Yu. A. Rozanov

Text	437
References	441

Multidimensional Stochastic Approximation

L. Schmetterer

1. Introduction	443
2. Notations	444
3. Some Auxiliary Results	444
4. Some Theorems	447
References	459

PART VI / Decision Procedures

Optimal Allocation of Observations when Comparing Several Treatments with a Control

Robert E. Bechhofer

1. Introduction and Summary	463
2. Statement of the Problem	464
3. An Alternative Expression for the Probability	465
4. Evaluation and Study of the Derivative $g'(\gamma)$	466
5. Summary of Results, and Fundamental Theorem	468
6. Barycentric Representation of the Optimal Allocation	470
7. Computation of the Optimal Allocation	471
8. Use of the Tables	472
9. Generalizations	473
References	473

Some Selection and Ranking Procedures for Multivariate Normal Populations

Shanti S. Gupta and S. Panchapakesan

1. Introduction and Summary	475
2. Selection with Respect to Multiple Correlation Coefficient	477
3. Selection of the Multivariate Normal Populations with Respect to Conditional Generalized Variance	495
4. Description of Tables	497
References	504

Decision Theoretic Approach to Some Multivariate Problems

Herman Rubin

Introduction	507
1. Inclusion of Variables in a Regression	508
2. "Scientific" Purpose of a Regression-Structural Inference	511
3. Discriminant Analysis	512
4. Factor Analysis	512
5. Looking at the Data	513

Selecting a Subset Containing at Least One of the t Best Populations

Milton Sobel

1. Introduction	515
2. Notation, Assumptions, and Requirement	516
3. Procedure R and Its PCS-Function	517
4. Infimum of the PCS under Procedure R	519
5. The Expected Subset Size for Procedure R ; Procedures R_0 and R'	521
6. Truncated Version of the RSS Procedures for $t > 1$	528
7. Numerical Illustrations	529
8. Asymptotic Efficiency of Procedure R relative to Procedure R_M	533
9. Related Problems	537
10. A Likelihood Ratio Procedure, R_L	539
References	540

PART VII / Econometrics, Principal Components, Reliability, and Applications

On the Structure and Estimation of General Interdependent Systems

Anders Ågren and Herman O. Wold

Introduction	543
1. Graphic Interpretation of Interdependent Systems in REID and GEID Specification	550
2. General Interdependent Systems	555
References	564

Exploratory Techniques Involving Artificial Variables

Rolf E. Bargmann

1. Introduction	567
2. Discriminant Analysis	568
3. The First Principal Component	573
4. Model Building Involving Latent Class Analysis	574
5. Noncentrality and Factor Analysis	577
6. Summary and Conclusion	579
References	580

On the Importance of Different Components in a Multicomponent System

Z. W. Birnbaum

1. Introduction: Definitions and Notations	581
2. Structural Importance	583
3. Reliability Importance	585
4. Structures with Modules	588
5. An Application	589
6. Concluding Remarks	591
References	591

Data Analytic Methods in Multivariate Statistical Analysis

R. Gnanadesikan and M. B. Wilk

Summary	593
1. Introduction	593
2. Reduction of Dimensionality	595
3. Multivariate Relationships	612
4. Multidimensional Classification	615
5. Assessment of Multivariate Statistical Models	621
6. Summarization and Exposure	624
7. Concluding Remarks	635
References	636

Geometrical Models and Badness-of-Fit Functions

Joseph B. Kruskal and J. Douglas Carroll

1. Geometrical Models and Badness-of-Fit Functions	639
2. Tree Structure Model	641
3. Parametric Mapping	644
4. Multidimensional Scaling	650
5. Multidimensional Unfolding	659
References	670

Optimality of Principal Components*Masashi Okamoto*

1. Introduction	673
2. Lemmas in Matrix Theory	674
3. Definition of Principal Components	679
4. Optimality of Principal Components	682
References	685

Information Theory and the Statistical Estimation of Econometric Relations*Gerhard Tintner and M. V. Rama Sastry*

Introduction	687
1. Theory of Information	687
2. Discriminant Function and Weighted Regression	689
3. Multiple Regression	690
4. Limited Information Maximum Likelihood	691
5. Two-Stage Least Squares	693
6. Multicollinearity and Use of Principal Components	694
References	695