

## CONTENTS

INTRODUCTION TO THE SERIES . . . . .	5
PREFACE . . . . .	7
LIST OF SYMBOLS . . . . .	15
<i>Chapter 1</i>	
INTRODUCTION . . . . .	17
1.1. The Markov probability model . . . . .	18
1.2. Estimation problems . . . . .	20
1.3. Plan of the book . . . . .	21
<i>Chapter 2</i>	
THE ESTIMATION OF TRANSITION PROBABILITIES FROM MICRO DATA . . . . .	23
2.1. The micro maximum likelihood (ML) estimator . . . . .	23
2.2. Bayesian analysis of the micro model . . . . .	25
a. Bayes' theorem and Bayesian inference . . . . .	26
b. The prior density function . . . . .	27
c. The posterior probability density function . . . . .	29
d. Bayesian estimation . . . . .	30
<i>Chapter 3</i>	
THE ESTIMATION OF TRANSITION PROBABILITIES FROM MACRO DATA . . . . .	31
3.1. A relation involving the macro data . . . . .	31
3.2. The unrestricted least squares transition probability estimator . . . . .	32
a. The row sum condition . . . . .	34
b. The $0 \leq p_{ij} \leq 1$ condition . . . . .	36
c. Some properties of the unrestricted estimator $\tilde{p}$ . . . . .	38
3.3. The restricted least squares transition probability estimator . . . . .	39
a. Sampling properties of the restricted estimator . . . . .	41

*Chapter 4*

THE SAMPLING EXPERIMENT AND SOME INITIAL RESULTS . . . . .	43
4.1. The simulated probability model and its characteristics . . . . .	44
4.2. The procedure of simulation . . . . .	46
4.3. The simulated population and sampling . . . . .	46
4.4. Sample proportions as the estimates of true proportions . . . . .	47
4.5. Basis for gauging estimator performance . . . . .	49
4.6. Experimental results for the maximum likelihood estimator using micro data . . . . .	51
4.7. Results from generated probability vector time series data . . . . .	54
4.8. Results from the sampling experiment macro data . . . . .	56
a. Unrestricted least squares . . . . .	56
b. Restricted least squares . . . . .	57
4.9. An application . . . . .	59
a. The brand change problem . . . . .	59
b. Results for the restricted and unrestricted estimators . . . . .	60

*Chapter 5*

WEIGHTED INEQUALITY RESTRICTED LEAST SQUARES ESTIMATORS . . . . .	63
5.1. The statistical model . . . . .	64
5.2. Weighted restricted least squares . . . . .	64
5.3. Some alternative weights . . . . .	66
5.4. Results from sampling experiment . . . . .	67
a. Weighted by average proportion of $i$ th state . . . . .	67
b. Weighted by an estimate of the $i$ th equation disturbance variance . . . . .	68
c. Weighted by product of average proportions in state $i$ . . . . .	68
5.5. Results for the brand change problem . . . . .	71

*Chapter 6*

A GENERALIZED LEAST SQUARES ESTIMATOR . . . . .	73
6.1. Non-spherical disturbances . . . . .	73
6.2. Redundant parameters and the reduced model . . . . .	75
6.3. Existence of the inverse of the disturbance covariance matrix . . . . .	76
6.4. Aitken's generalized unrestricted and restricted least squares estimators . . . . .	79
6.5. Results from the sampling experiment . . . . .	81

*Chapter 7*

THE MINIMUM CHI-SQUARE ESTIMATOR . . . . .	85
7.1. Preliminaries . . . . .	85
7.2. The restricted minimum chi-square estimator . . . . .	86

7.3. The modified restricted minimum chi-square estimator . . . . .	87
7.4. An equivalent model . . . . .	88
7.5. A numerical example . . . . .	90
 <i>Chapter 8</i>	
THE MACRO MAXIMUM LIKELIHOOD ESTIMATOR . . . . .	93
8.1. The multinomial distribution under the Lexis scheme . . . . .	93
8.2. The mode of the likelihood function . . . . .	95
8.3. The macro maximum likelihood estimator . . . . .	99
8.4. Results from the sampling experiment . . . . .	102
8.5. Some applications . . . . .	102
a. Results for the brand change problem . . . . .	102
b. Results for the tenure status problem . . . . .	105
 <i>Chapter 9</i>	
BAYESIAN ANALYSIS OF THE 'MACRO' MODEL . . . . .	107
9.1. Bayesian analysis: prior distribution . . . . .	107
9.2. Likelihood function and posterior distribution . . . . .	109
9.3. The mode of the posterior distribution . . . . .	110
9.4. Comparison with some sampling theory results . . . . .	112
9.5. A macro Bayesian transition probability estimator . . . . .	115
9.6. The Bayesian approach: further considerations . . . . .	117
9.7. A numerical example . . . . .	119
9.8. Sampling experiment results . . . . .	120
a. Multi-beta leptokurtic prior . . . . .	121
b. Multivariate beta leptokurtic prior . . . . .	123
c. Results from a platykurtic prior . . . . .	125
9.9. Results for the brand change problem . . . . .	127
 <i>Chapter 10</i>	
THE MINIMUM ABSOLUTE DEVIATIONS ESTIMATOR . . . . .	131
10.1. Specification of the statistical model . . . . .	131
10.2. Linear programming formulation . . . . .	132
a. Minimizing the unweighted sum of the absolute deviations . . . . .	132
b. Minimizing the weighted sum of the absolute deviations . . . . .	133
10.3. Results from the sampling experiment . . . . .	134
 <i>Chapter 11</i>	
PREDICTION AND THE CHI-SQUARE GOODNESS-OF-FIT TEST . . . . .	139
11.1. Predicted proportions . . . . .	139
11.2. The chi-square goodness-of-fit test . . . . .	141
11.3. Results from sampling experiment . . . . .	142

*Chapter 12*

COMPARISONS OF THE ALTERNATIVE ESTIMATORS . . . . .	145
12.1. The basis for comparison . . . . .	145
12.2. Aggregate mean square error and variance measure . . . . .	145
12.3. Wilcoxon matched-pairs signed-ranks test and Kendall's coefficient of concordance . . . . .	150
12.4. Summary . . . . .	156

*Chapter 13*

CONCLUDING REMARKS . . . . .	157
------------------------------	-----

*Appendix A*

THE GENERALIZED INVERSE METHOD . . . . .	163
A.1. A generalization of the generalized least squares . . . . .	163
A.2. The generalized inverse of the disturbance covariance matrix for the Markov probability model . . . . .	168
A.3. The multicollinearity case . . . . .	173
A.4. Row sum condition and the reduced weight matrix . . . . .	174
A.5. The uniqueness of the generalized inverse estimator is the Aitken's generalized least squares with redundant parameters deleted . . . . .	178
A.6. Summary . . . . .	179

*Appendix B*

THE GENERAL LINEAR PROBABILITY MODEL . . . . .	183
B.1. The model . . . . .	183
B.2. The unrestricted estimator . . . . .	185
B.3. The restricted estimator . . . . .	186
B.4. A joint estimation procedure . . . . .	190

*Appendix C*

ESTIMATION OF VARIABLE TRANSITION PROBABILITIES . . . . .	191
C.1. The model . . . . .	191
C.2. The unrestricted estimator . . . . .	195
C.3. The restricted estimator . . . . .	196
C.4. Concluding remarks . . . . .	197

*Appendix D*

THE FORTRAN PROGRAMMING OF CLASSICAL AND BAYESIAN TRANSITION PROBA- BILITY ESTIMATORS . . . . .	199
D.1. The standard procedure . . . . .	199

D.2. Incorporation of the prior knowledge with sample observations . . .	200
D.3. Deleting a column . . . . .	200
D.4. Assignment of weight . . . . .	201
D.5. Recursive solutions for the ML and Bayesian estimators . . . . .	201
D.6. The use of the control 'DITTO' and 'DIT.' . . . .	202
D.7. The use of the control 'CLEAR' and 'SUMMARY' . . . . .	202
D.8. Option control card . . . . .	203
D.9. An input example . . . . .	205
D.10. An output example . . . . .	206
D.11. Fortran listing of computer routine . . . . .	209
REFERENCES . . . . .	243
AUTHOR INDEX . . . . .	251
SUBJECT INDEX . . . . .	253