Contents

PREFACE xi
NOTATION xiii

Chapter 1 **Introduction and Certain Double Sampling Procedures**

1.1 Introduction to Sequential Testing 1
1.2 Sampling Inspection 3
1.3 Loss and Risk Functions 6
1.4 Certain Double Sampling Procedures 7

Chapter 2 **The Sequential Probability Ratio Test**

2.1 The Sequential Probability Ratio Test (SPRT) 14
2.2 Finite Termination of the SPRT 23
2.3 The Operating Characteristic Function (OC Function) 25
2.4 Average Sample Number 28
2.5 Wald's Fundamental Identity 40
2.6 Bounds for the Expected Sample Size in a SPRT 54
2.7 Formulas for Calculating the Operating Characteristic and the Average Sample Number of Some Sequential Tests 67
2.8 Truncated SPRT 76
2.9 Optimal Properties of the SPRT 78
2.10 Generalized SPRT (GSPRT) 87

2.11 Restricted SPRT 89
2.12 Extended SPRTs 93
2.13 Asymptotic Properties of SPRTs 97

Chapter 3 **Sequential Tests for Composite Hypotheses**

3.1 Wald's (1947) Method of Weight Functions 101
3.2 Sequential t- and t^2-Tests and Sequential Analogues of Stein's Two-Stage Test 103
3.3 Sequential F-Test 119
3.4 Certain Two-Sample Sequential Procedures 121
3.5 Fraser Sufficiency and Conditional Test 123
3.6 Invariant Sequential Procedures 126
3.7 Sequential Likelihood Ratio Test Procedures 147
3.8 Sequential Tests between Three Hypotheses 163
3.9 Sequential k-Decision Problems 175
3.10 Sequential Tests for Two-Sided Alternatives 195
3.11 Efficiency of the SPRT 209
3.12 Bayes Sequential Procedures 227
3.13 Iterated Logarithm Inequalities and Their Application to Sequential Testing Hypotheses 244
3.14 Certain Nonparametric Sequential Test Procedures 253
3.15 Locally Most Powerful (LMP) Sequential Tests 284

Chapter 4 **Sequential Estimation**

4.1 Introductory Remarks 293
4.2 Certain Two-Stage Procedures 293
4.3 Formulation of the Problem of Sequential Estimation by Intervals or Sets 315
4.4 Wald's Sequential Estimation Procedures 316
4.5 Certain General Concepts of Sequential Estimation 317
4.6 Cramér–Rao Lower Bound for the Sequential Case 317
4.7 Sufficiency and Completeness in the Sequential Case 322
4.8 Minimax Estimation 332
4.9 A Class of Situations Where Bayes Sequential Estimation Procedure Is Nonsequential 343
4.10 Sequential Estimation and Certain Closed Sequential Decision Procedures 358
4.11 Estimation by Double Sampling 367
4.12 Large Sample Theory of Sequential Estimation 377
4.13 Sequential Confidence Intervals for the Mean of a Normal Population with Unknown Variance 385
4.14 The Asymptotic Theory of Fixed-Width Sequential Confidence Intervals for the Mean 412
4.15 Asymptotic Theory of Fixed-Size Sequential Confidence Bounds for Linear Regression Parameters 428
4.16 Confidence Sequences for Other Parameters 435
4.17 Sequential Estimation of the Mean Vector of a Multivariate Normal Distribution 452

4.18 A General Method of Determining Fixed-Width Confidence Intervals 464
4.19 Sequential Estimation of the Size of a Finite Population 470
4.20 Consistency of Certain Sequential Estimators 475
4.21 Asymptotically Optimal Bayes Sequential Estimation 482
4.22 Sequential Nonparametric Confidence Intervals for Certain Parameters of the Population 487
4.23 Certain Problems of Optimal Stopping 504

Appendix 1 **Solution to Wald's Equation** 513

Appendix 2 **Differentiation of an Expectation** 515

Appendix 3 **On the Moments of a Random Variable** 518

Appendix 4 **The Normal Diffusion Process** 520

Appendix 5 **Large Sample Properties of Maximum Likelihood Estimates** 527

Appendix 6 **A Table of Standard Distributions** 530

Appendix 7 **A Useful Convergence Theorem of Cramér** 532

Appendix 8 **Backward Induction** 534

REFERENCES AND AUTHOR INDEX 537
SUBJECT INDEX 555